

# The emergence of scaling in QCD jets

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Follow-up: can this fact be made useful?

## Outline

- Introduction: Multi-jet final states
- Jet scaling in data/theory
- The origins of jet scaling
- Jet vetos and other applications
- Conclusions/Speculation

Based on:

EG, Schumann; To appear

EG, Plehn, Schumann; PRL 108.032003, 2012; hep-ph/1108.3335

Englert, EG, Plehn, Schichtel, Schumann; hep-ph/1110.1043

Also:

Englert, Plehn, Schichtel, Schumann; Phys.Rev.D83:095009,2011; hep-ph/1102.4615

Englert, Plehn, Schichtel, Schumann; hep-ph/1108.547

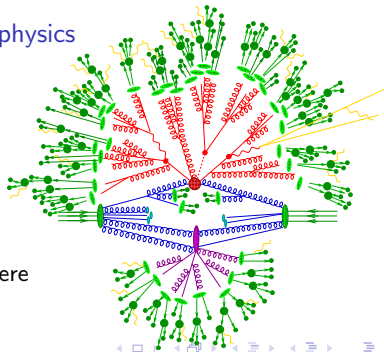
## Motivation for understanding multi-jet final states

### LHC as a QCD machine

- The goal of the LHC is to explore new physics in and beyond the SM
- Every analysis at the LHC requires understanding jets [especially events without jets!]
- High-multiplicities crucial for SM measurements [top properties...]
- Multi-jets are background for BSM searches [SUSY cascades, new colored states, black holes, ...]

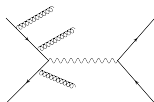
### QCD as a messy environment for new physics

- Initial and final states are non-perturbative objects (hadrons)  
[CTEQ, NNPDF; Fragmentation functions]
- Hard process and subsequent parton shower understandable in terms of perturbative QCD
- but...even making good predictions here can be a challenge



## The challenge for theoretical predictions at the LHC

### Calculational techniques for multi-jets



- Leading order calculations suffer from scale uncertainty
- Difference between two (equally good) scale choices  $\mu$  and  $\bar{\mu}$

$$\sigma_{n\text{-jets}}^{\text{LO}}(\bar{\mu}) - \sigma_{n\text{-jets}}^{\text{LO}}(\mu) \sim \alpha_S^n \left( n b_0 \alpha_S \ln \frac{\bar{\mu}^2}{\mu^2} \right)$$

- NLO calculation less scale sensitive but not always available [pure jets  $n \leq 4$  for leading color,  $n \leq 3$  at full NLO] [Hoeche et al. Blackhat]
- Also, there are selections/processes/observables with large logarithms

$$\alpha_S \log \left( \frac{Q}{Q_0} \right) \sim 1$$

- Analytic techniques to resum large logarithms available [but limitations]



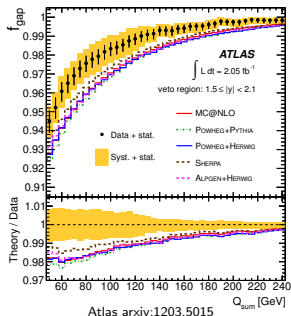
# The challenge for theoretical predictions at the LHC

## Examples of theoretical tools pushed to their limit

- MonteCarlo Matrix-Element/Parton-Shower methods flexible tools including some LO, NLO and logarithmic effects
- Good general agreement between early data and MonteCarlo methods
- But...inherent limitations on PS evolution is the largest uncertainty in many analysis. [i.e. signal modelling MC@NLO vs. POWHEG]

## Jet scaling

- New tools for understanding multi-jet events always welcome.
- The idea of jet scaling may be one of them but...in what observable do we look



## Jet ratios as a handle on scaling and multi-jet rates

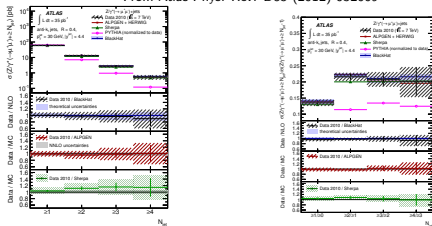
$\sigma_n$  is the exclusive  $n$  jet cross section (in addition to core process jets)

$$R_{n+1/n} \equiv \frac{\sigma_{n+1}}{\sigma_n}$$

## Why are jet ratios a convenient observable for study?

- Experimentally: systematics tend to cancel.
- Theoretical: scale uncertainties also tends to be weaker
- Visually: easy to interpret and much easier to see patterns [see next slide]

From Atlas Phys. Rev. D85 (2012) 032009



## Observed Scaling Patterns

### Staircase [Steve Ellis, Kleiss, Stirling (1985); Berends

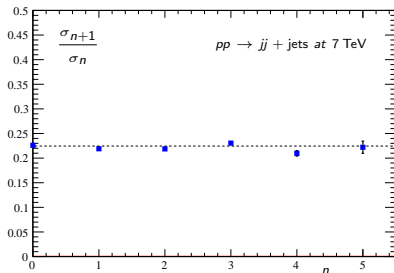
(1989)]

- Ratios are constant

$$\sigma_n^{\text{exclv}} \sim R_0^n \equiv e^{-bn}$$

$$\Rightarrow \frac{\sigma_{n+1}}{\sigma_n} = e^{-b} = R_0$$

- Observed: UA1, Tevatron, LHC



### Poisson [Peskin & Schroder; Rainwater, Summers,

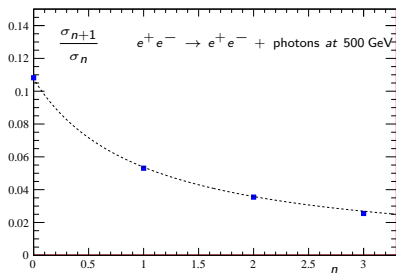
Zeppenfeld (1997) ]

- Ratios are not constant

$$\sigma_n^{\text{exclv}} \sim e^{-\bar{n}} \bar{n}^n / n!$$

$$\Rightarrow \frac{\sigma_{n+1}}{\sigma_n} = \frac{\bar{n}}{n+1}$$

- Observed: e.g. photons at LEP



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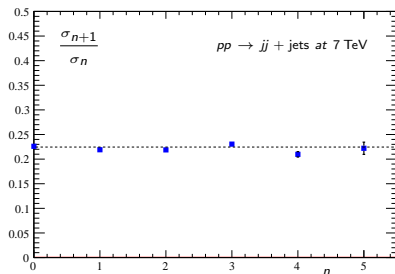
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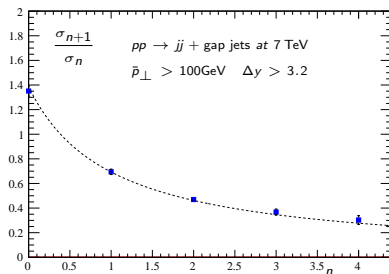
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# Jet scaling in the data

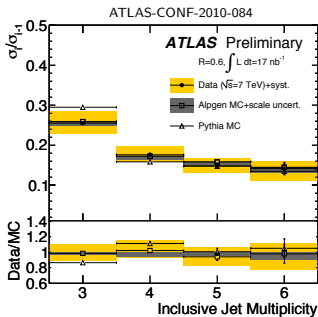
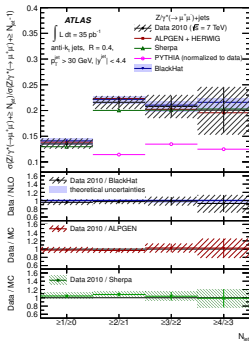
## Finding patterns in current analysis

- Both patterns observed in QCD processes (depending on cuts)

- For staircase:  $R_{excl} = R_{incl}$  [Englert, Plehn, Schichtel, Schumann]

- For Poisson:

$$R_{excl} = \frac{\bar{n}}{n+1} \iff R_{incl} = \left( \frac{(n+1) e^{-\bar{n}} \bar{n}^{-(n+1)}}{\Gamma(n+1) - n\Gamma(n, \bar{n})} + 1 \right)^{-1}$$



## QED and the emergence of Poisson scaling

### Basic synopsis of Poisson radiation pattern from QED [Peskin & Schroder; Weinberg]

- Fully factorized form of the matrix element (Eikonal approximation)
- Phase space factor  $1/n!$  for identical bosons in the final state

$$\Rightarrow \sigma_n \sim \frac{L^n}{n!} e^{-L} \quad \text{with} \quad L \sim \frac{\alpha}{\pi} \log \left( \frac{E_{hard}}{E_{soft}} \right)$$

### Crucial theorem: Addition of independent Poisson processes

- Suppose two Poisson processes  $N_1$  and  $N_2$  with Poisson expectations  $\bar{n}_1$  and  $\bar{n}_2$  are independent. The counting process  $N$  defined by  $N(t) = N_1(t) + N_2(t)$  is a Poisson process with rate function  $\bar{n}$  given by  $\bar{n} = \bar{n}_1 + \bar{n}_2$ .

$\Rightarrow$  All QED processes give Poisson [in soft collinear limit]

## Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit
2. QCD has (logarithmically equivalent) subsequent splittings (gluon 3-pt)
3. Different kinematics due to PDFs

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⇒ Corrections due to hard matrix elements

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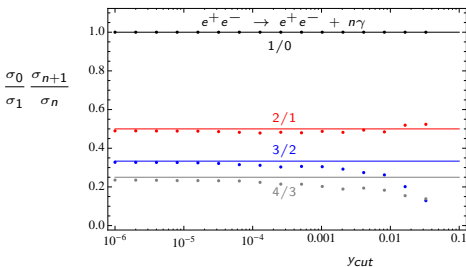
## Corrections due to the hard matrix element

### Require photons to be widely separated

- Full amplitude contains finite terms [only log terms exponentiate]
- De-emphasize log terms by scanning final state with Durham measure; *i.e.* only keep particles with

$$2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij}) > y_{cut} s.$$

- Destroys the Poisson scaling, but cannot be responsible for a staircase
- Ratios are pushed in the wrong direction (not surprising due to phase space)



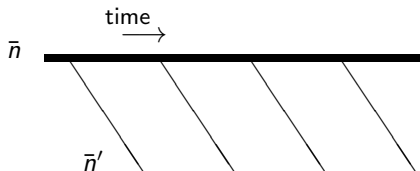
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1. QED Poisson scaling is derived in the soft-collinear limit  
⇒ ~~Corrections due to hard matrix elements~~
2. QCD has (logarithmically equivalent) subsequent splittings (gluon 3-pt)  
⇒ Secondary emissions
3. Different kinematics due to PDFs

## Statistical model for an iterated Poisson process

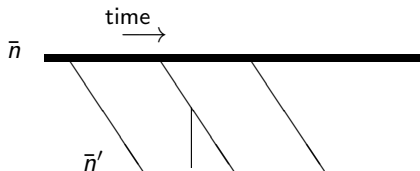
- Extension of the (pure Poisson) exponentiation model [Rainwater, Zeppenfeld]
- Simple analogy; each emission generates a new Poisson process with separate Poisson parameter  $\bar{n}'$



$$P(4, \bar{n}, \bar{n}') \sim e^{-\bar{n}-4\bar{n}'} \frac{\bar{n}^4}{4!}$$

## Statistical model for an iterated Poisson process

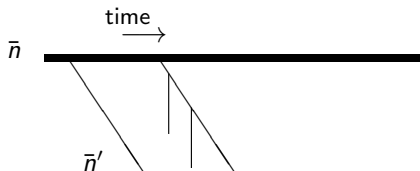
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$$P(4, \bar{n}, \bar{n}') \sim e^{-\bar{n}-4\bar{n}'} \frac{\bar{n}^4}{4!} + \frac{1}{2} e^{-\bar{n}-4\bar{n}'} \bar{n}^3 \bar{n}'$$

## Statistical model for an iterated Poisson process

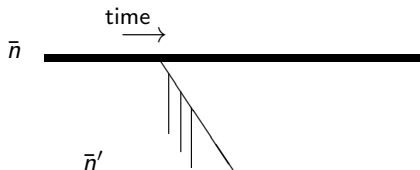
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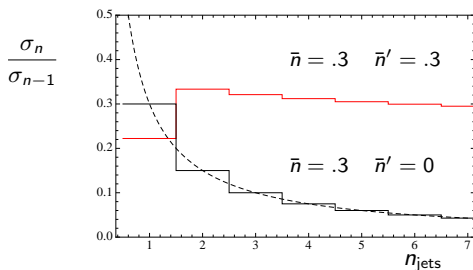
## Statistical model for an iterated Poisson process

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## Statistical model for an iterated Poisson process

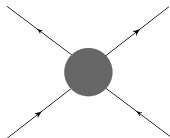


- Limit of “gluon” dominated ( $\bar{n}' \gg \bar{n}$ ) evolution gives flat staircase scaling
- Also computable result at LL for pure gluon initiated jet [Konishi, Veniziano]
- Iterated Poisson process can compute exact jet rates for Leading-Log

$$\bar{n} \sim C_F \int_{Q_0}^Q \frac{dz}{z} \log \frac{Q}{z} \quad \bar{n}' \sim C_A \int_{Q_0}^Q \frac{dz}{z} \log \frac{Q}{z} \int_{Q_0}^z \frac{dz'}{z'} \log \frac{Q}{z'}$$

## Parton shower in a nutshell

1. Generate core process with set of outgoing particle momenta

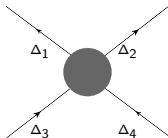




## Parton shower in a nutshell

1. Generate core process with set of outgoing particle momenta
2. Attach to each external line Sudakov form factor [no-splitting probability]

$$\Delta^i(q) = \exp \left\{ - \int_{Q_0}^q dt \Gamma^i(q, t) \right\} .$$

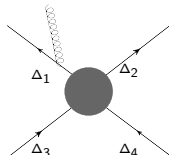


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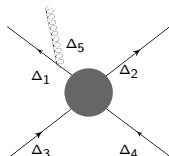


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3. Integrate virtuality  $q$  from hard scale to hadronization scale
4. With each splitting, attach a new Sudakov and repeat



## Final state parton cascade

1. All radiative emissions start off as a Poisson process [expand "core-process" Sudakov]

$$\Delta_q = 1 - \Gamma_q + \frac{1}{2}\Gamma_q^2 + \dots = \begin{array}{c} \diagup \\ \diagdown \end{array} + \begin{array}{c} \diagup \\ \diagdown \\ \text{---} \end{array} + \begin{array}{c} \diagup \\ \diagdown \\ \text{---} \\ \text{---} \end{array} + \dots$$

2. Secondary emissions break Poisson scaling [e.g.  $e^+e^- \rightarrow \text{jets}$ ]

$$\begin{array}{c} \diagup \\ \diagdown \\ \text{---} \\ \text{---} \end{array} \sim \Gamma_q \otimes \Gamma_g$$

3. Relative size of primary and secondary splitting processes give us  $\bar{n}/\bar{n}'$

$$\bar{n}/\bar{n}' \sim \begin{array}{c} \diagup \\ \diagdown \\ \text{---} \\ \text{---} \end{array}$$

## Subsequent splittings and the emergence of staircase

More realistic model:  $e^+e^- \rightarrow q\bar{q} + \text{jets}$ 

- Leading log and next-to-leading log jet rates available for the Durham measure (we calculate to  $\mathcal{O}(\alpha^4)$ ) [Catani, Dokshitzer, Olsson, Turnock, Webber (1991); Webber (2010)]

$$L \equiv \log \frac{1}{y_{\text{cut}}} \quad \text{and} \quad a \equiv \alpha_S/\pi$$

- Purely abelian terms from  $qg$  splitting exponentiate

$$f_2^D = 1 - a \frac{C_F}{2} L^2 + a^2 \frac{C_F^2}{8} L^4 - a^3 \frac{C_F^3}{48} L^6 + a^4 \frac{C_F^4}{384} L^8$$

$$f_3^D = a \left( \frac{C_F}{2} \right) L^2 - a^2 \left( \frac{C_F^2}{4} + \frac{C_F C_A}{48} \right) L^4 + a^3 \left( \frac{C_F^3}{16} + \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{960} \right) L^6 - a^4 \left( \frac{C_F^4}{96} + \frac{C_F^3 C_A}{384} + \frac{C_F^2 C_A^2}{1920} + \frac{C_F C_A^3}{21504} \right) L^8$$

$$f_4^D = a^2 \left( \frac{C_F^2}{8} + \frac{C_F C_A}{48} \right) L^4 - a^3 \left( \frac{C_F^3}{16} + \frac{C_F^2 C_A}{48} + \frac{7C_F C_A^2}{2880} \right) L^6 + a^4 \left( \frac{C_F^4}{64} + \frac{C_F^3 C_A}{128} + \frac{C_F^2 C_A^2}{512} + \frac{C_F C_A^3}{5120} \right) L^8$$

$$f_5^D = a^3 \left( \frac{C_F^3}{48} + \frac{C_F^2 C_A}{96} + \frac{C_F C_A^2}{720} \right) L^6 - a^4 \left( \frac{C_F^4}{96} + \frac{C_F^3 C_A}{128} + \frac{3C_F^2 C_A^2}{1280} + \frac{41C_F C_A^3}{161280} \right) L^8$$

$$f_6^D = a^4 \left( \frac{C_F^4}{384} + \frac{C_F^3 C_A}{384} + \frac{7C_F^2 C_A^2}{7680} + \frac{17C_F C_A^3}{161280} \right) L^8$$

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- Non-abelian terms do not simply exponentiate!

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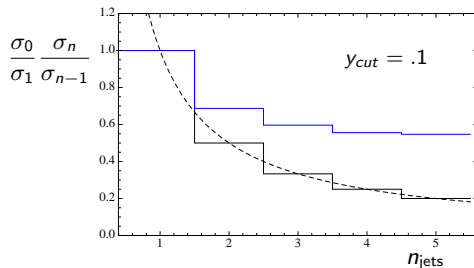
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## Statistical model for an iterated Poisson process

- Leading log jet rate ratios in  $e^+e^- \rightarrow$  jets flatter than Poisson [higher multiplicities]
- Still not enough to completely explain the data in Drell-Yan





## Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

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~~Corrections due to hard matrix elements~~

2. QCD has (logarithmically equivalent) subsequent splittings (gluon 3-pt)

⇒ Secondary emissions ✓

3. Different kinematics due to PDFs

⇒ Relative cost of an additional jet depends on previous jets

## Estimating the effect due to PDFs

- The ratio of exclusive cross-sections contains a ratio of PDF's evaluated at different typical scales

$$R_{(n+1)/n} \sim \frac{f(x_{n+1}, Q_{n+1})}{f(x_n, Q_n)}$$

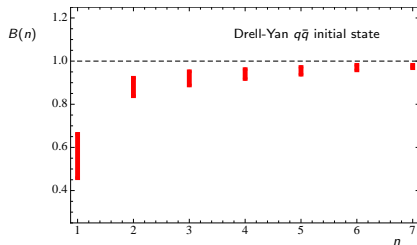
- Measure of the suppression on exclusive multiplicities given by the (discrete) 2nd-derivative with respect to  $x$ .

$$B(n, Q) = \frac{|f(x_{n+1}, Q_{hard})|^2}{f(x_n, Q_{hard}) f(x_{n+2}, Q_{hard})}$$

- Two representative kinematic extremes
  1.  $Z$  recoils against jets
  2. Each jet costs moves  $x$  by  $\delta x = p_{\perp} / \sqrt{s}$
- Choose factorization scale below the jet scale [exclusive jet rates]

## Estimating the effect due to PDFs

Lowest bin most affected and effect decreases with multiplicity



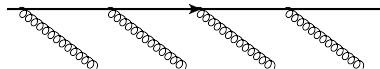
## Important differences between QED and QCD

Deviation from Poisson must be the result of one of the following

1. QED Poisson scaling is derived in the soft-collinear limit  
⇒ ~~Corrections due to hard matrix elements~~
2. QCD has subsequent splittings via gluon 3-point vertex  
⇒ Secondary emissions ✓
3. Different kinematics due to PDFs  
⇒ Relative cost of an additional jet depends on previous jets ✓

## Other correlations affecting Poisson scaling

- Color correlation at sub-leading  $N$

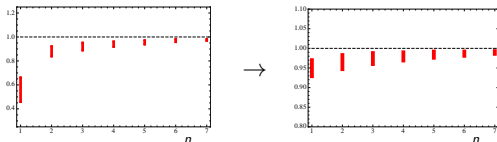


- Next-to-leading-log contributions and interference effects
- Not strictly Sudakov type logarithms [BFKL, HEJ, ...]
- Finite corrections to the hard matrix element

## The return to Poisson scaling in QCD

Observation: Poisson distribution returns when we require a hard final state jet

- PDF effect disappears as we move to higher  $x$  [example Drell-Yan]



- Large logarithm drives evolution along a single line [explains why we do not see Poisson scaling in color singlet exchange]
- Primary (abelian) emission off the hard color line are dynamically enhanced [see generating functional formalism]
- Note that inducing a large initial state logarithm (e.g. cut on the lepton invariant mass) is not sufficient to induce the Poisson behaviour [again fixed energy jet rates imply this]

## Summary slide on the origins of scaling patterns

1. Jet ratios show two scaling patterns:

Poisson or staircase

2. Poisson scaling is a fundamental result which can most easily be seen by expanding the Sudakov form factor off a given hard line
3. Staircase scaling is not a fundamental consequence of QCD but a fortuitous semi-coincidence of two effects:

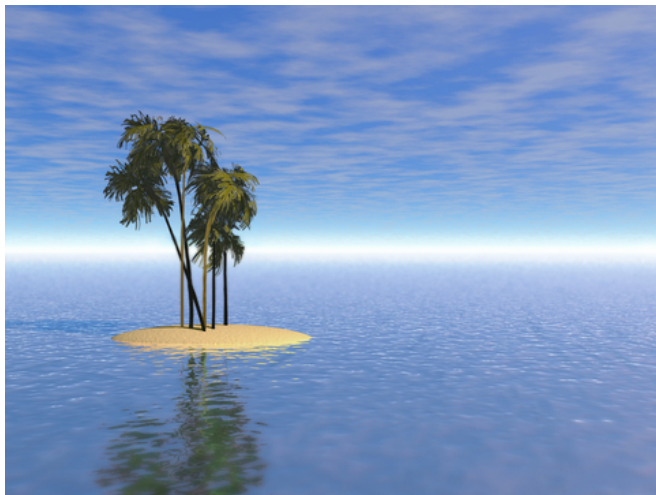
secondary emissions and PDF suppression .

4. We can undo both effects by imposing a large final state logarithm, and return to Poisson scaling

# Applications



## Deserted island physics



Deserted island physics: calculate the (normalized) Drell Yan  $N_{jet}$  ratios

## Modern way

1. Find favorite parton shower MC
2. Wait a while (couple of days?)
3. Compute each  $N_{jet}$  cross-section
4. Divide rates to obtain ratios

## Deserted island way (scaling arguments)

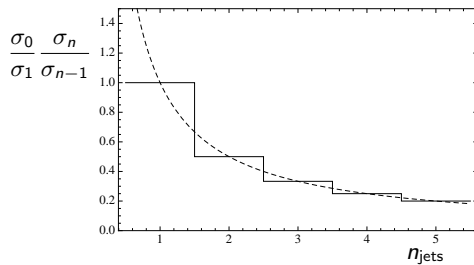
1. Everything starts as a Poisson
2. Add 1st order inhomogeneity [from  $g \rightarrow gg$  splitting functions]

$$\bar{n} \sim 1 \qquad \bar{n}' \sim \frac{C_A}{12C_F}$$

3. Evaluate PDF function B
4. Fold together!

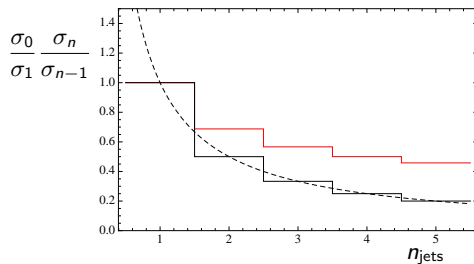
## The origin of jet scaling patterns (Illustration)

1. All radiative emissions start off as a Poisson process



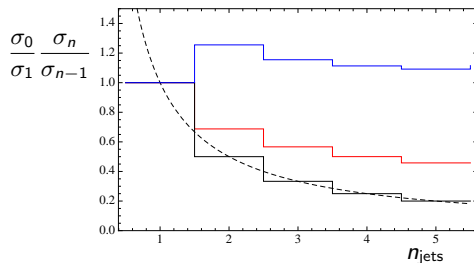
## The origin of jet scaling patterns (Illustration)

1. All radiative emissions start off as a Poisson process
2. Secondary emissions break Poisson scaling [model non-homogenous Poisson process]

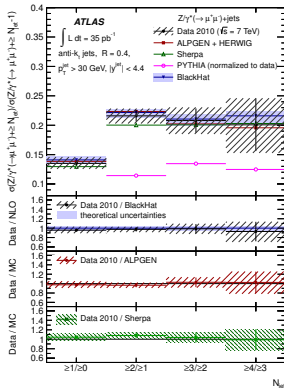
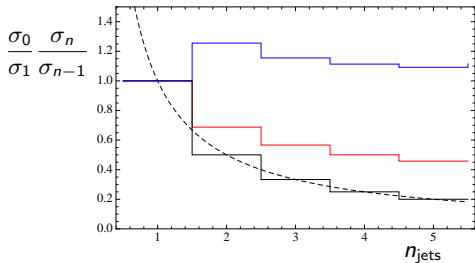


## The origin of jet scaling patterns (Illustration)

1. All radiative emissions start off as a Poisson process
2. Secondary emissions break Poisson scaling [model non-homogenous Poisson process]
3. PDF kinematics mean that cost of producing additional jets is  $n$  dependent



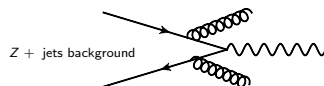
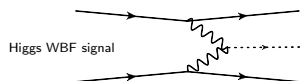
## The origin of jet scaling patterns (Illustration)



## Jet Vetos in Higgs searches

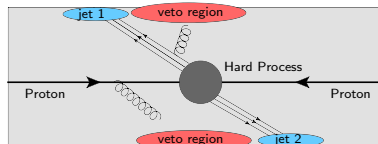
### Higgs searches via Weak Boson Fusion

- Early sensitivity in  $H \rightarrow WW^*/\tau\tau/\gamma\gamma$  [both ATLAS and CMS dedicated WBF searches]
- Large background from  $V(V) + \text{jets}$  at  $\mathcal{O}(\alpha_s^2)$



### Central Jet Veto [for how QCD can actually help us for once!]

- Widely separated (in  $\eta$ ) tagging jets and veto “in-between” QCD activity
- Signal gives less soft in-between QCD radiation [Rainwater, Zeppenfeld]
- $N_{jet}$  distribution give us valuable information on central jets



## Predicting Central Jet veto efficiencies

### Weak Boson Fusion cuts

1. Identify tagging jets with  $|y_1 - y_2| > 4.4$  and  $m_{jj} > 600$  GeV
2. Veto events with an additional central jet satisfying  $p_{\perp} > 20$  GeV
3. Define the 2-jet exclusive cross section for signal and background

### Theoretical difficulties in the WBF jet veto computation

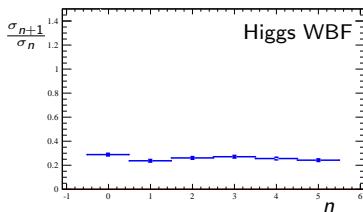
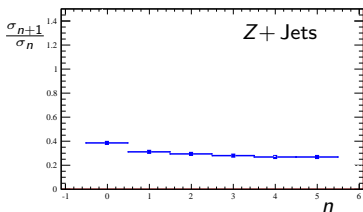
- The second step induces a large logarithm  $\sim \log \left( \frac{p_{\text{veto}}}{p_{\text{tag}}} \right)$
- Cannot trust fixed order calculations (single emission probability  $> 1$ )
- Analytic resummation available in some cases [Delgado, Forshaw, Marzani, Seymour]
- The only general method involves Parton-Shower [preferably with matrix element matching]



Validation of theoretical tools in  $H \rightarrow \tau\tau$  [EG, Plehn, Schumann]

## Before Weak Boson Fusion cuts

- WBF and  $Z$ + jets background follow a flat staircase distribution



## Veto Probabilities

- Can use Poisson fit to calculate veto probability taking into account all multi-jet contributions.

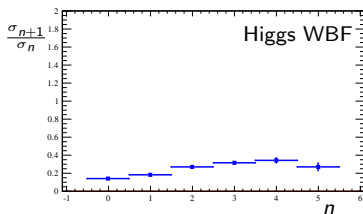
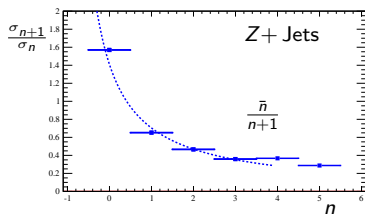
$$P_{\text{veto}} = e^{-\bar{n}}$$

- Many (data rich) processes available for comparison [QCD gap fraction,  $Z/W/\gamma$  + jets]

Validation of theoretical tools in  $H \rightarrow \tau\tau$  [EG, Plehn, Schumann]

## After Weak Boson Fusion cuts

- $Z$ +jets background quickly becomes Poisson, while WBF does not



## Veto Probabilities

- Can use Poisson fit to calculate veto probability taking into account all multi-jet contributions.

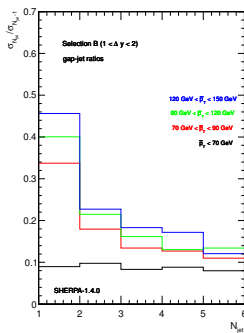
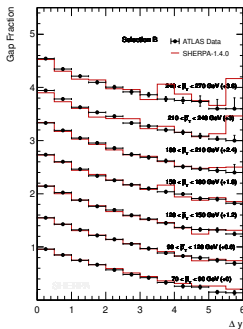
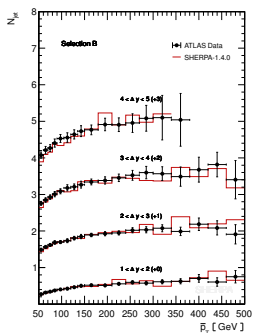
$$P_{\text{veto}} = e^{-\bar{n}}$$

- Many (data rich) processes available for comparison [QCD gap fraction,  $Z/W/\gamma$  + jets]

## Validation of tools and scaling hypothesis

Atlas study on jets between gaps as a function of  $p_{\perp}$  and  $\Delta y$ 

based on RIVET public-analysis from ATLAS JHEP 1109 (2011) 053

Same for  $Z + jets$  analysis/figures

## Conclusions

- Ratio of  $N_{jet}$  distributions interesting from a theory perspective and crucial for precise predictions for jet vetos.
- Many opportunities to study the Poisson distribution in other  $n_{jet}$  distributions, for example
  - ▶  $Z/\gamma + jets$  with a large leading jet  $p_{\perp}$ .
  - ▶  $Z + jets$  in WBF type of configurations (large  $\eta$  separations), with large average jet  $p_{\perp}$  of leading two jets.
  - ▶ Pure QCD di-jets with a large rapidity gap.
  - ▶  $t\bar{t} \rightarrow b\bar{b}WW^* \rightarrow l\nu l\nu + jets$ ?
- The fact that QCD cross-section do not at all follow a Poisson distribution (unless a large final state logarithm is present) is the result of a combination of secondary splittings and PDF effects.
- Theoretical origins for scaling patterns mostly understood. Full quantitative study requires resummed jet rates.
- We suspect more Pheno applications...and we're working on this!

## Central jet vetos in Higgs searches [EG, Plehn, Schumann]

### Weak Boson Fusion

- Signal has high  $|\eta|$  tagging jets,  $Z + \text{jets}$  background more likely to radiate additional jets into this region. Impose WBF cuts

$$y_1 \cdot y_2 < 0 \quad |y_1 - y_2| > 4.4 \quad m_{jj} > 600 \text{ GeV}$$

- Central jet veto makes this channel relevant. Veto all events with an additional jet satisfying

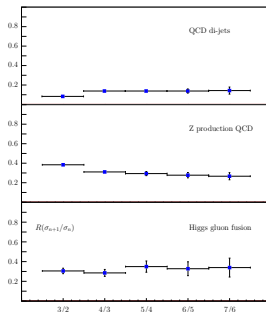
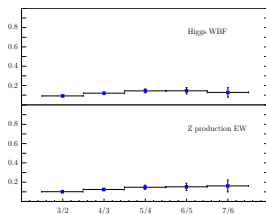
$$p_T^{\text{veto}} > 20 \text{ GeV} \quad \min y_{1,2} < y^{\text{veto}} < \max y_{1,2}$$

- For background, Dipole initiated shower contains a large Logarithm.

$$\bar{n} \sim \log \frac{m_{jj}}{Q_{\text{veto}}}$$

- For signal, large log not induced.

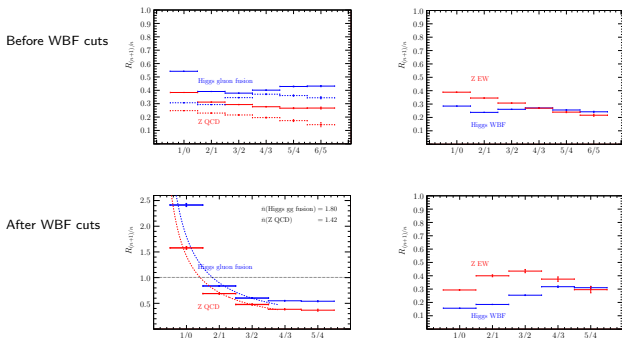
Before WBF cuts



Validation of theoretical tools in  $H \rightarrow \tau\tau$  [EG, Plehn, Schumann]

## The emergence of Jet scaling patterns

- Through simulation we see that WBF cuts induce Poisson scaling [in background]



- Our standard  $Z/W/\gamma + \text{jets } N_{jet}$  ratios
- For both distribution we can easily compute the survival probability

$$\sigma_2^{\text{exclusive}} = \sigma_2^{\text{inclusive}} e^{-\bar{n}}$$

## The origin of Jet scaling patterns

How can we get a handle on non-exponentiable contributions?

- Inhomogenous Poisson processes [each emission emits]

$$P(n, \bar{n}, \bar{n}') = e^{-\bar{n}-n\bar{n}'} \sum_{i=0}^n \left( \frac{(n-1)!}{i!(n-i-1)!(n-i)!} \right) \bar{n}'^i \bar{n}^{n-i}$$

Deviation from Poisson to 1st order  $\iff$  compute  $\bar{n}'$

How can we reasonably estimate PDF effect?

1. Construct “threshold” kinematics for process and cuts (e.g. Drell-Yan)
2. Compute discretized second derivative

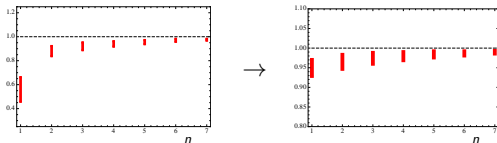
$$B = \frac{|F(x_{n+1}, Q)|^2}{F(x_n, Q)F(x_{n+2}, Q)},$$

3. Multiply ratios by  $B$

## The return to Poisson scaling

Observation: Poisson distribution returns when we require hard final state jets

- PDF effect disappears as we move to higher  $x$  [example Drell-Yan]



- Hard final state jet drives evolution along single line [explains why we do not see Poisson scaling in color singlet exchange]
- Exponentiable wrt core process emission off the hard color line are dynamically enhanced [see generating functional formalism]
- Return to fully factorized form of the matrix element



## Missing energy searches and laboratories for studying jet scaling

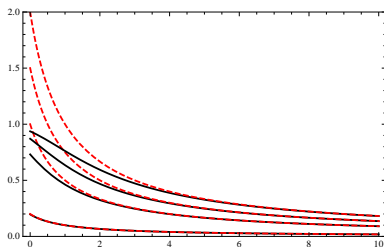
### Photons + jets and missing $E_T$ searches [Englert , Plehn, Schumann, Schichtel ]

- Ratio of  $N_{jet}$  distributions interesting from a theory perspective and crucial for precise predictions for jet vetos.
- Ratio of  $N_{jet}$  distributions interesting from a theory perspective and crucial for precise predictions for jet vetos.

## More on Poisson

## Experimental Observation

$$R_{excl} = \frac{\bar{n}}{n+1} \quad R_{incl} = \left( \frac{(n+1) e^{-\bar{n}} \bar{n}^{-(n+1)}}{\Gamma(n+1) - n\Gamma(n, \bar{n})} + 1 \right)^{-1}$$



## Summary slide on the origins of scaling patterns

### Poisson scaling

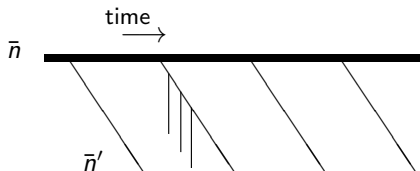
- Poisson scaling is a fundamental result for the soft collinear limit of massless gauge theories [most easily seen by expanding the Sudakov form factor evolving a given hard line]
- Emissions contained in this expansion wrt the core process generate a Poisson distribution while secondary emission break the scaling pattern.
- We expect a Poisson distribution to answer question such as, "what is the multiplicity distribution of 10GeV photons at LEP" or jets accompanying the process in a  $Z + 1$  jet, where the jet has  $p_{\perp} > 30\text{GeV}$ .

### Staircase (or Berends scaling)

- Not a fundamental consequence of QCD radiation, but rather a fortuitous semi-coincidence of two effects which both act to flatten the ratio distribution
- We expect this type of behavior for processes and selections not driven by a single large final state logarithm, so that instead primary and secondary emission are roughly equally likely.

## Leading-log QCD as a statistical process

- For the double-leading-logs in QCD; only physics is in choosing  $\bar{n}$  and  $\bar{n}'$
- Compared with the generating functional formalism, this is a dramatic simplification



$$P(4, \bar{n}, \bar{n}') \sim e^{-\bar{n}-4\bar{n}'} \frac{\bar{n}^4}{4!} + \frac{1}{2} e^{-\bar{n}-4\bar{n}'} \bar{n}^3 \bar{n}' + \frac{3}{2} e^{-\bar{n}-4\bar{n}'} \bar{n}^2 \bar{n}'^2 + e^{-\bar{n}-4\bar{n}'} \bar{n} \bar{n}'^3$$