Paul Bourke

Grouse, Hurricanes and Dead Cats: How to predict

Peter Watson

Prediction and Time

Even if we cannot time-travel, perhaps we can do it "virtually"

if we know exactly how a system works & how it changes with time, we should be able to predict its future.

What can we predict?

Deterministic systems: i.e. systems whose future can be predicted exactly e.g. planetary system, mass on a spring, pendulum.

Random systems: i.e. ones which are too complex to predict exactly e.g. gas, society...Best we can do is to predict average values

However there are two other kinds of systems:

Chaotic: i.e. systems which are predictable over the short term but not over the long term.

Quantum: systems which are intrinsically unpredictable except in a special sense.

Chaotic Motion

She comes, she comes, the sable throne behold
Of Night Primeval and of Chaos old!
...
Physic of Metaphysic begs defence
And Metaphysic calls for aid on Sense
See Mystery to Mathematics fly
In vain! they gaze, turn giddy, rave and die
....
Lo! thy Dread Empire, Chaos is restored
Light dies before thy uncreating Word.
Alexander Pope, The Dunciad

"baker transform"

Take a piece of dough with a raisin

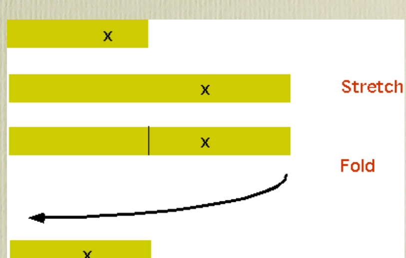
Stretch it to twice it's original length

Fold it in half

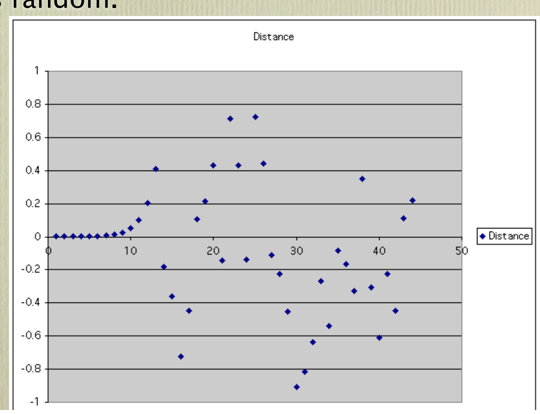
Where is the raisin?

The formula is

$$x_n = 2x_{n-1} \quad (x_{n-1} < .5)$$
$$x_n = 2 - 2x_{n-1} \quad (x_{n-1} > .5)$$



Start with two raisins very close together and see what happens:
If you plot the difference in their positions , it looks nice and smooth to start with, but suddenly becomes random.



in Arcadia, Valentin wonders why the population of grouse on the moors isn't predictable.
Suppose we have a population which grows

No. of births ∝ No of live individuals

$$x_{birth} = kx_{live}$$

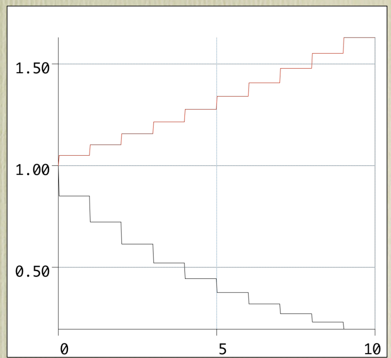
No. of deaths ∝ no. of live individual

$$x_{death} = k' x_{live}$$

So total number in next generation is

$$x_{n+1} = x_n - x_{death} + x_{birth}$$
$$= (1 + k - k')x_n$$

If this is all, population grows (or dies!) exponentially.



But suppose we add in starvation

- In that case if the population grows too large, there will be starvation, and this deaths will increase more rapidly: say as square of the population

$$x_{starve} = k'' x_{live}^2$$

- So total number in next generation is

$$x_{n+1} = x_n - x_{death} - x_{starve} + x_{birth}$$
$$= (1 + k - k')x_n - k'' x_n^2$$

In Practice

- Suppose k = .3, k' = .15, k" = .001

#	Born	Die	Starve	Next #
100	30	15	10	105
200	60	30	40	190

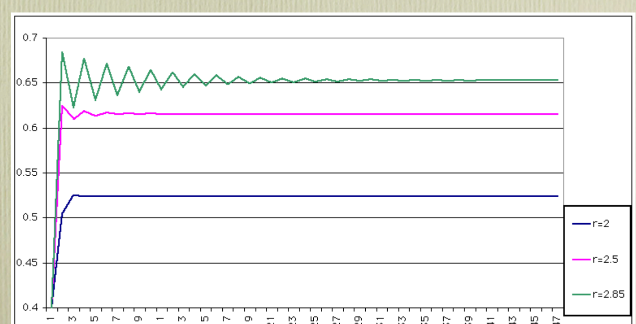
- " Obviously " what will happen is that the population will grow until the population reaches an equilibrium value?

150	45	22	23	150

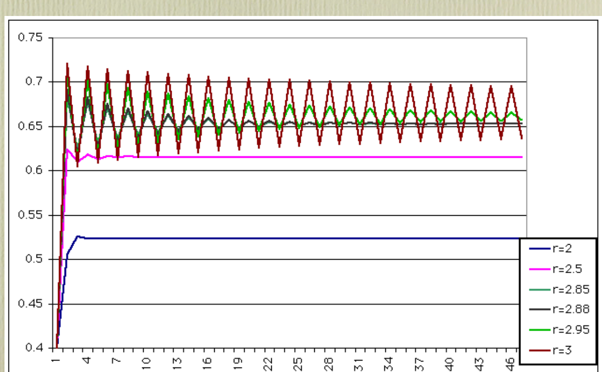
We can make this look a bit cleaner by writing

$$x_{n+1} = rx_n(1 - x_n)$$

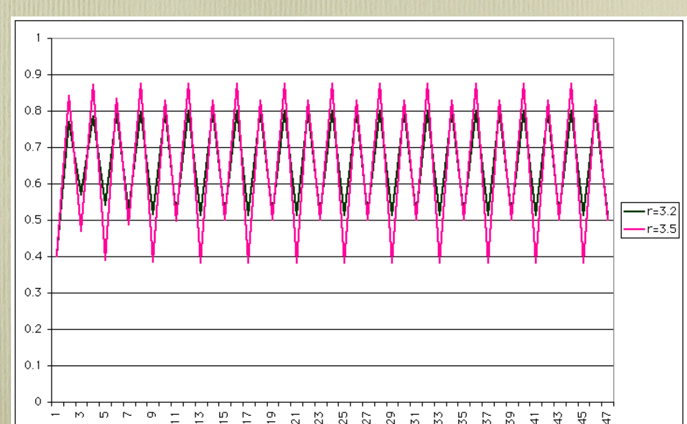
Deaths = Births (but there will be a bit of overshoot)



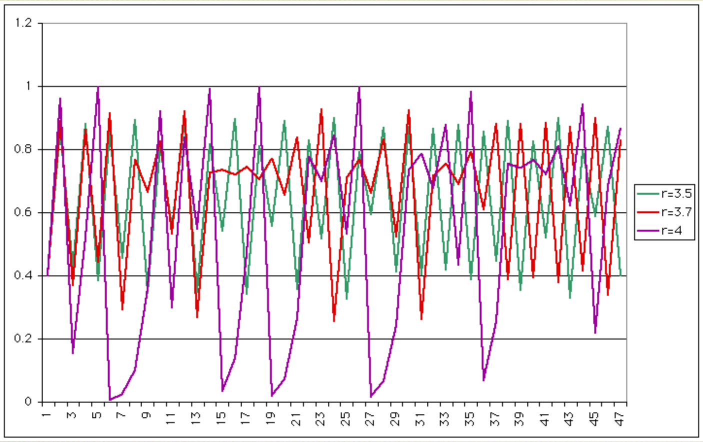
- But then the “overshoot” doesn’t die away and the system oscillates



- But then it gets worse



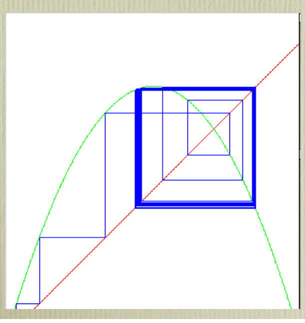
- and worse

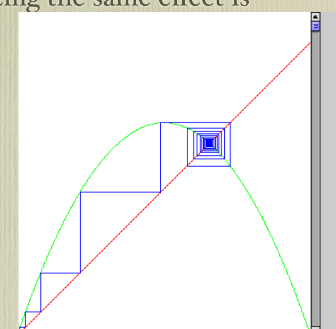


Logistic Map

- VALENTINE "You have some x-and-y equations. Any value for x gives you a value for y. So you put a dot where it's right for both x and y. Then you take the next value for x which gives you another value for y, and when you've done that a few times you join up the dots and that's your graph of whatever the equation is....every time she works out a value for y, she's using that as her next value of x. And so on." Arcadia

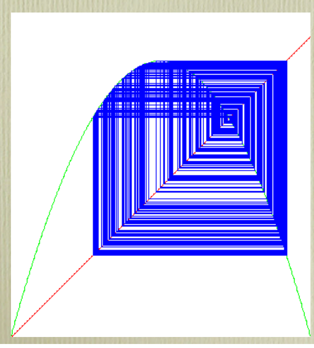
- Another way of visualizing the same effect is the "logistic map"

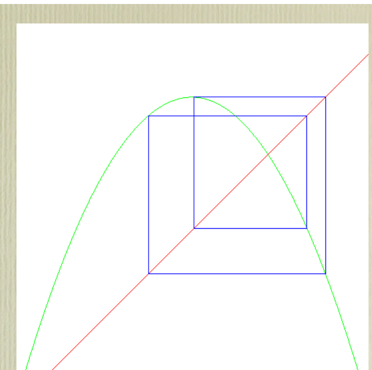




But increase r (the green curve) a bit and it doesn't settle down

Add a bit more and it goes into a 4-cycle





- Until things get really chaotic

Chaotic Systems

All chaotic systems have some common features

The equations must all be non-linear: i.e. Have terms like x^2

There are regions of the parameters where the motion is predictable

There are regions where it is chaotic

In the chaotic region, points that start off close together become wildly different as time goes on.

e.g. http://math.bu.edu/DYSYS/applets/index.html

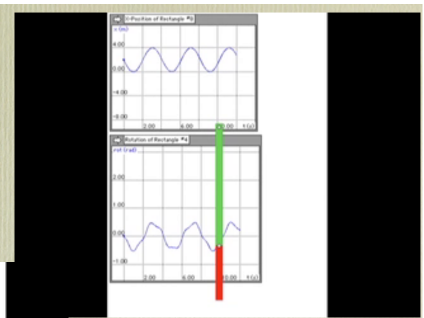
Note the importance of non-linearity!

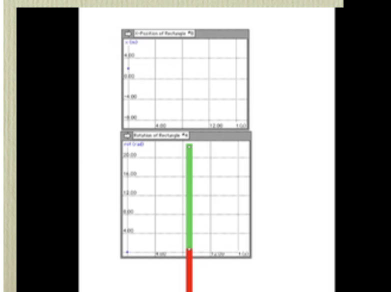
- Linear systems can be unmapped

Double pendulum

- Small swings are predictable,

Medium swings are quasi-periodic





Large ones are chaotic

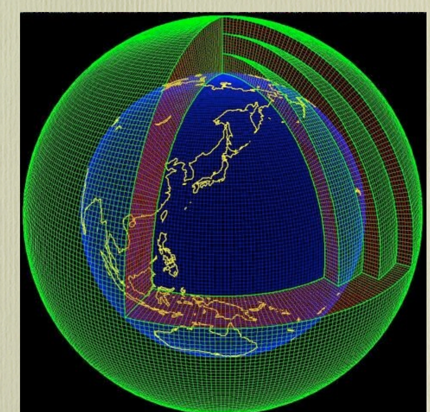
Weather

- "Primitive Equations" for weather written down by L F Richardson (1922). Can't be solved without computer

"After so much hard reasoning, may one play with a fantasy? Imagine a large hall like a theatre, except that the circles and galleries go right round through the space usually occupied by the stage. The walls of this chamber are painted to form a map of the globe. The ceiling represents the north polar regions, England is in the gallery, the tropics in the upper circle, Australia on the dress circle and the antarctic in the pit.
A myriad computers are at work upon the weather of the part of the map where each sits, but each computer attends only to one equation or part of an equation. The work of each region is coordinated by an official of higher rank. Numerous little "night signs" display the instantaneous values so that neighbouring computers can read them. Each number is thus displayed in three adjacent zones so as to maintain communication to the North and South on the map.

- (computers in old fashioned sense of someone who computes!)

This is how we do it



But

The "Lorentz" equations: a very simplified version of the equations that describe weather. These give rise to chaotic behaviour: hence it is assumed that weather itself is chaotic.

Butterfly effect: arbitrarily small perturbation of initial conditions have unpredictably large consequences

Weather is also chaotic

You cannot predict the future weather precisely.

However, buried in this are some predictable elements. e.g. we cannot predict an "el Nino" event, but we can predict the consequences once it has happened.

Note "weather" prediction and "climate" prediction are (almost) unrelated

•Can predict globally, not locally
Can predict how fast a river will flow

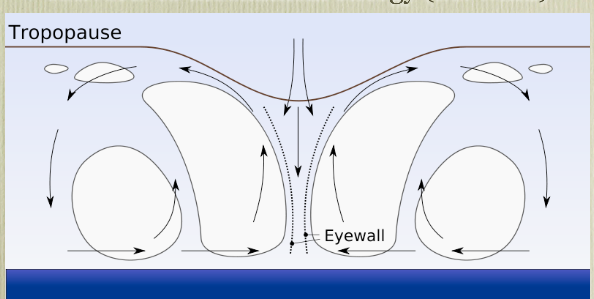
- But not how it will behave on small scale

PW

Hurricanes

NASA picture

- Driven by same set of processes
- Warm water in Caribben is easy to evaporate,
- energy transferred from ocean to upper atmosphere
- converts to mechanical energy (i.e. wind)

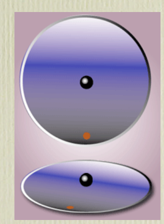


NASA

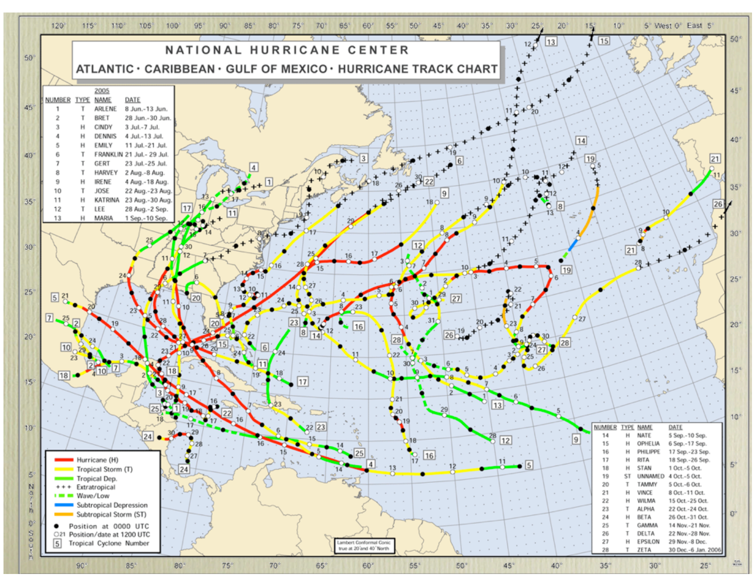
- Energy release ~ 10^{20} J/day
- power is 1PW = 10^{15} W ~ 100 times total power consumption of humanity

Hurricanes rotate anti-clockwise and drift west & north because of Coriolis force

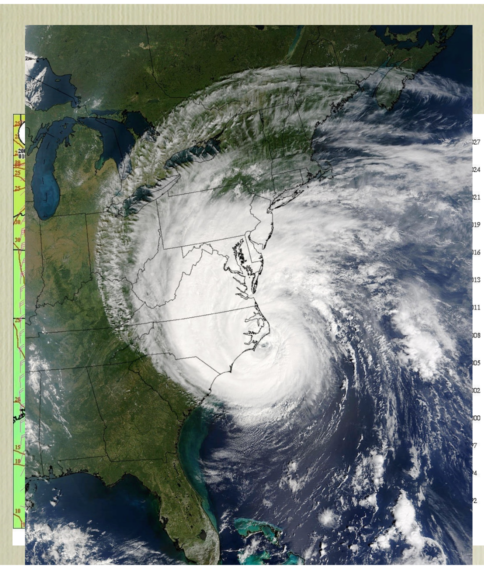
Earth rotates, so it is a non-inertial frame of reference



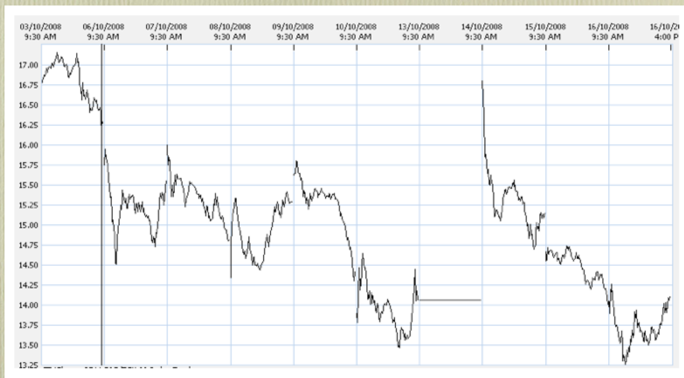
Wikisource



Can do it over the short term Hurricane Isabelle



An interesting chaotic system (provided your pension doesn't depend on it!)

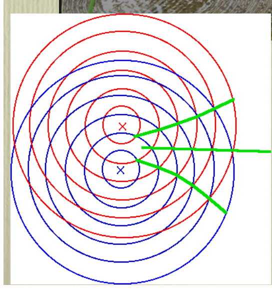


- At the start of the crisis financial firms held huge dollops of each others equity..Such tight coupling increases the danger of "non-linear" outcomes, where a small change has a big impact. Economist Feb 2010

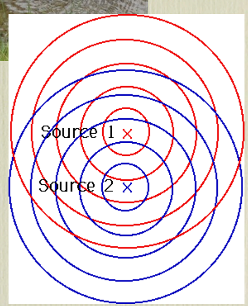
Now we do the hard stuff

- Quantum Mechanics
- I think I can safely say that nobody understands quantum mechanics. (Richard Feynman.)

PW,



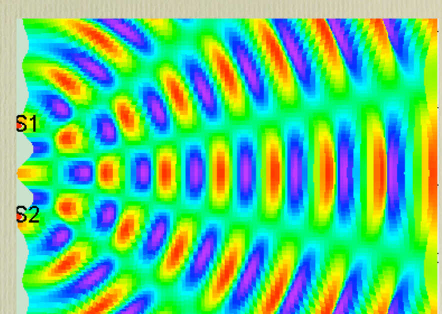
there are places where the crests add up



Ripple tank

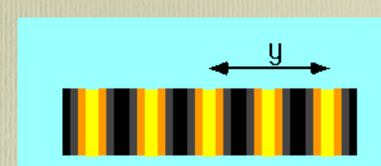
PW

- Like this



Wikisource

- When you do this for light, you get
- bright bands (adding up)
- dark bands (cancelling out)



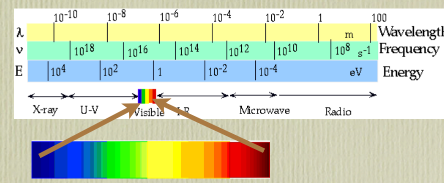
Direct Demonstration that light is a wave (also lets you find λ)

And the most important thing we learn is from barbecues

- What's hot and what's not: roughly
- red is 800°C
- orange is 1500°C
- yellow is 2000°C
- blue is 15000°C
- X-rays are 1 million °C

Text

- Light is part of the whole electromagnetic spectrum



- All waves satisfy $f\lambda = c$
- (frequency ×wavelength = speed)
- Since they have a frequency, they are a clock!

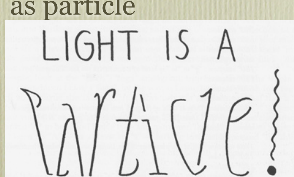
- But suppose light is a particle...
- Planck (1900) suggested that E.M. radiation is emitted in lumps of energy (quanta) which became known as "photons"
- First we need new unit of energy: Joule is much too large
- 1 electron-volt (eV) = 1.6×10^{-19} J
- 0.00000000000000000016 J
- most chemical processes involve energies of a few eV per molecule

PW

- Energy of photon = Planck's constant x frequency
- $E = hf$
- Photons are also particles with a difference:
- Always travel at c (speed of light) and can easily be created and destroyed.

But you told me light was a wave............! What is light?

- Particle? Newton, Descartes
- Wave? Young, Huyghens
- Yes? Planck, Einstein
- Light travels as wave, but arrives and departs as particle



Douglas R. Hofstadter

What is light?

Particle? Newton, Descartes
Kerner: Look at the edge if the shadow. It is straight like the edge of the wall that makes it. This means light is ..little bullets. Bullets go straight.
Hapgood (Tom Stoppard)

Wave? Young, Huyghens
Kerner: When you shine a light through two little gaps, side by side, you don't get particle patterns like for bullets, you get wave patterns like for water. The two beams of light mix together
Hapgood (Tom Stoppard)

Yes? Planck/Einstein
Light travels as wave, but arrives and departs as particle

Wave-Particle Duality
De Broglie (1924)

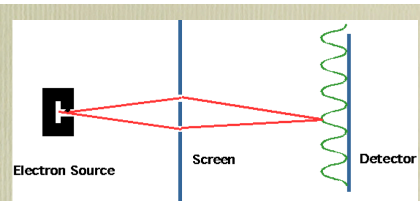
- You cannot ask:
- Is light a wave or a particle? answer is “yes”
- so maybe electron (particle) has some wave properties.....

PW

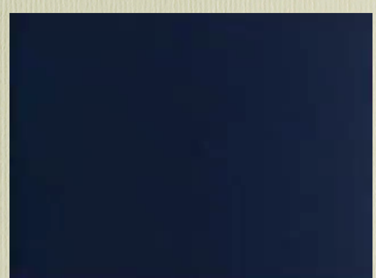
- What is wave-length of electron?
- de Broglie guessed for an electron wavelength $\lambda = h/mv$
- if v = 1000 ms^{-1}, ➠ λ = 500 nm (like yellow light!)

PW

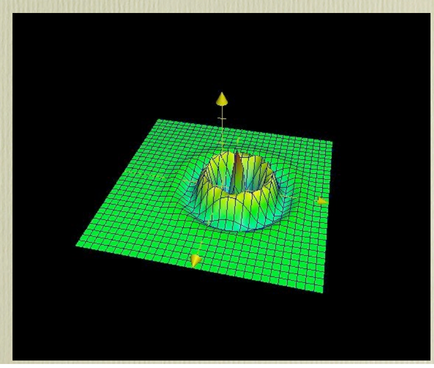
- We can now do this with electrons: Very low energy electrons pass through slits and hit detector (e.g. photo plate) and give 2-slit interference pattern



You can even watch how it builds up, one electron at a time



- Wave - particle duality:
- All fundamental (i.e small!) particles also act like waves (what is an electron?...)
- waves act like particles.
- or a wavicle!



PW

- With the (in principle) simple assumption that waves have particle-like properties and particles have wave-like properties, we can understand all of the problems that arose at the turn of the century.
- But this is only part of quantum mechanics: we can also understand

Antiparticles: For every particle with given properties, there is a corresponding anti-particle with the properties flipped:
- e.g. electron has charge -1.6×10^{-19} C
- positron has same mass, charge = 1.6×10^{-19} C

Solids and liquids: e.g why copper is a good conductor and plastic is a lousy one
Nuclear forces (why don't they simply fall apart, what makes uranium radio-active, but not lead)
Transistors and hence integrated circuits
Light in fibres
Stars (how long will the sun last,and what will happen to it
Superconductors (why some materials conduct electricity perfectly)
Lasers (another idea that started with Einstein)
Magnetic Resonance Imaging (MRI)

Since quantum mechanics works so well, maybe we shouldn't worry about what it actually means.....

- But we have some problems:
- **Which slit did the electron go through?**

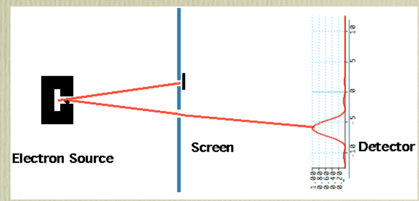
We choose to examine a phenomenon which is impossible, **absolutely impossible**, to explain in any classical way, and which has in it the heart of the quantum mechanics. In reality it contains the only mystery...Any other situation in QM, it turns out, can always be explained by saying, "You remember the case of the experiment with the two holes? It's the same thing."

Richard Feynman, the Character of Physical Law

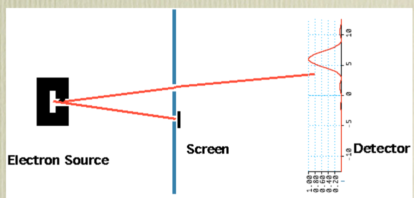
What Waves?

- The electron is a particle, with charge. It must go through one slit or the other...

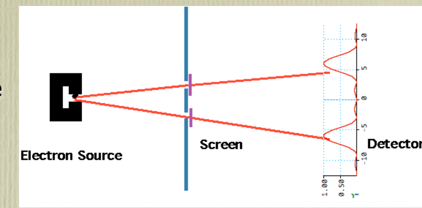
Suppose we close off one slit:



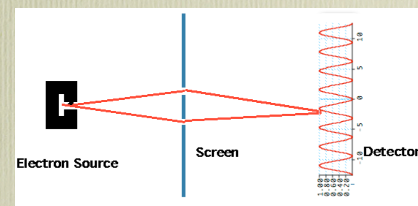
Suppose we close the other slit:



When we add together two one slit patterns, We get this



Not what we get from 2-slits together



- Suppose we get sneaky and allow electron through but check which slit it went through.

Now we get sum of one slit patterns, but not a 2 slit pattern!

What happens if we use a detector that only picks up one electron in two?

More worrying than this: we can do a "delayed choice" experiment: don't try to observe the electron until **after** it has gone through one of the slits...that still destroys the pattern.

Conclusion **We cannot decide which slit the electron went through without destroying the pattern. Observing something fundamentally changes it!**

There was a young man who said "God
Must think it exceedingly odd
That this tree
Continues to be
When there's no one about in the Quad"

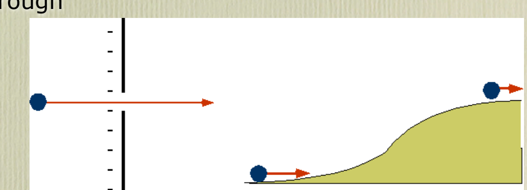
Kerner: Now we come to the exciting part. We will watch the bullet to see how they make waves ...The wave pattern has disappeared
Because we looked. Every time we don't look, we get wave pattern. Every time we look to see how we get wave pattern we get particle pattern
Hapgood (Tom Stoppard)

So why should you care, since this is a lecture about Time?

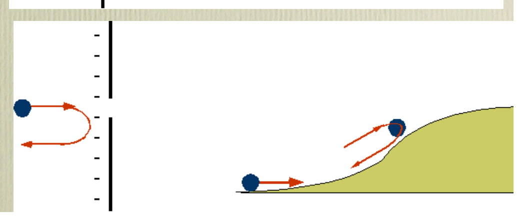
- Because we cannot say what happened **after** it happened!
- I can only say, there we have been: but I cannot say where.
And I cannot say, how long, for that is to place it in time.
T. S. Eliot (Burnt Norton)

What Waves?

- Obvious interpretation: electron is the wave.
- Electron is like a tiny particle: if it hits a barrier it either goes through

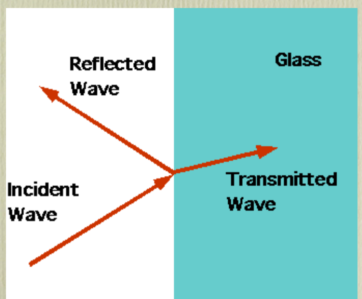


or gets reflected if the energy is too low

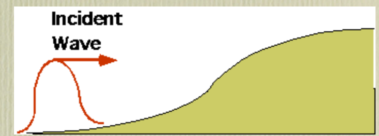


What Waves?

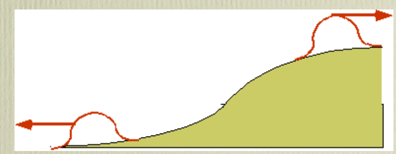
- When waves hit a barrier, they get partially reflected (like light hitting glass).



If electron is literally the wave,



This would imply we see 1/2 electrons



But we don't!

Probability Interpretation

- Wave represents probability of particle being at given place: more precisely

Note Electron must be somewhere: i.e. probability of detecting it somewhere = 1

Think of a die:

probability of any given face = 1/6

probability of any face being uppermost = 1

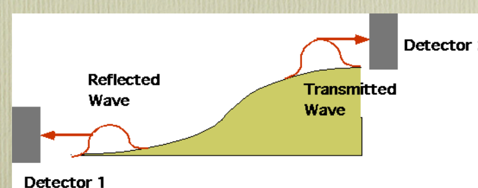
Back to barrier problem

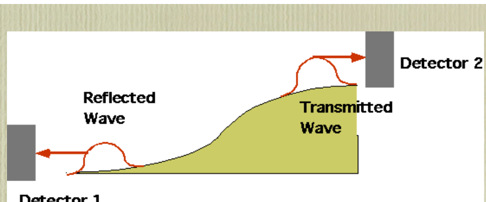
Probs must add to 1:

P_1 = prob. that electron hits detector 1:

P_2 = prob. that electron hits detector 2

$P_1 + P_2 = 1$





If (say) P_1 = .5 and we fire 1000 electrons,

- ◦ 481 could hit 1
- ◦ 519 ------------ 2

- (Maybe)
- 1000 will hit 1 or 2
- But we **cannot** say what any individual electron will do

Classical Determinism

Given state of solar system in (say) 100 A. D., can use Newtonian mechanics to predict earth's position now

Quantum mechanics:

Can only predict most likely (probable) position now.

Morals

1. Macroscopic (i.e. large) objects are predictable, electrons aren't!
2. Cannot ask "what happens?": can only ask "what can we measure?"
3. No reason to assume that rules deduced for macroscopic objects are true for very large/very light/ very fast objects.
4. "What colour is an electron?"

Measurement

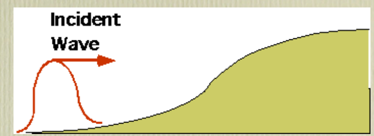
In classical mechanics, we believe that a object is the same whether we measure it or not.

In quantum mechanics, until we have measured it, its condition is indeterminate.

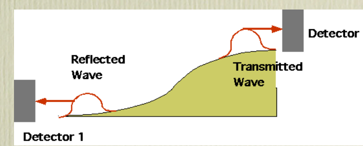
- E.g.: suppose we measure the position of a particle and it was here →C
- Where was it just before?
- **Classical Mechanic** At C.
- **Quantum Mechanic** Somewhere: it was only measuring it that fixed its position . Where is a candle flame after it is blown out?

Have we given free will to the electron?

- E.g. go back to our wave function example:



- This seemed to say that the electron gets split in half, but we interpreted it as a probability.
- But when did the electron decide which way it was going?



- **Classical Mechanic** Obviously at the moment it was reflected.

  Quantum Mechanic It is indeterminate until you measure it

- The Einstein-Podolsky-Rosen paradox (EPR) is a more sophisticated version of this

  God does not play dice. Einstein

  Hence the only logical way out is "hidden variables": underneath quantum mechanics, there is some "clockwork" where everything is deterministic.

  it only looks random on the surface.

Schrödinger's Cat

was supposed to show the idiocy of people who really believed in quantum mechanics.

- The trivial version: you have a box, with a lid: when it is opened, cyanide gas is released.

  Take a cat.

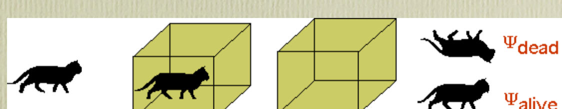
  Put it in the box and close the lid.

  Is the cat dead?

  Why don't you look?

- The sophisticated version: you have a box, with a lid and a single radioactive atom: when the atom decays, cyanide gas is released.

  Take a cat
  Put it in the box and close the lid.
  Is the cat dead or alive?



- **Classical Mechanic** Obviously its either dead or alive
- **Quantum Mechanic** It is indeterminate until you measure it . More exactly, the cat is a mixture of alive and dead cats: the measurement fixes it.
- **Schrödinger** Don't be stupid.

Both Einstein and Schrödinger were wrong.

Bell's theorem shows that there is a measurement that you can do on the polarizations of the particles which is incompatible with any possible hidden variable theory.

Aspect did the experiment.

The Schrödinger's Cat experiment has been done:

No animals were injured in the making of this movie.

One atom: process is totally random, so you can't decide if a one-atom cat is alive or dead without measuring it(!)

Many atoms (10^{29}): constitutes an independent measuring system, so the cat measures it's own deadness

Few atoms (2-20): process becomes steadily more predictable

God not only plays dice, but throws them where they cannot be seen.
Hawking

Measurement

- This “measurement fixes things” is known as the “Collapse of wave function”: obviously very ugly .

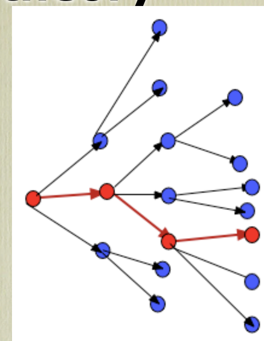
How does the electron know it is being measured?.

Do we need an actual conscious observer?

Is there a link between consciousness and QM?

Many worlds theory

Many-worlds theory: Everett (1957) . Every time a measurement is made, the universe subdivides into separate universes that correspond to every possible outcome



“You're in the right place and this is the right time, but I'm afraid you're in the wrong alternate universe.”

Avoids observation problems, but not testable (?) and not very economical!

In all fictional works, each time a man is confronted with several alternatives, he chooses one and eliminates the others; in the fiction of Ts'ui Pên, he chooses-simultaneously-- all of them. He creates in the diverse way, diverse futures..which themselves also proliferate and fork. *The Garden of Forking Paths, Borges.*

What might have been is an abstraction
Remaining a perpetual possibility
Only in a world of speculation.
What might have been and what has been
Point to one end, which is always present.
Footfalls echo in the memory
Down the passage which we did not take
Towards the door we never opened
Into the rose-garden.
T. S. Eliot (Burnt Norton)

- We can calculate measured values with phenomenal accuracy
- E.g. An electron acts like a tiny magnet: exactly how tiny?
- In sensible units
- -1.001159652181 (2006 measured)
- -1.001159652182 (2008 theory)
- So quantum mechanics cannot be ***wrong***

Conclusions:

Either Quantum mechanics is correct, and there is no "simpler" system

Or Reality is even uglier than we thought: e.g.

non-local hidden variables: every bit of the universe is involved with every other bit: very Zen, but totally wipes out free will!

????????????

(Ugh!) Does it bother you that 20th century technology depends fundamentally on something no-one understands?

Where does this leave prediction?

- Predictions (especially of the future) are hard!
- We can PROVE some very simple systems with exact equations are unpredictable
- We (more-or-less) understand what systems are predictable
- Some very complex systems ARE partly predictable
- Quantum systems allow very accurate average predictions but no individual predictions
- And seem to forbid retrodictions!