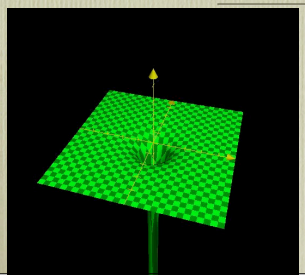
Going Straight in a Bent Space: How Matter bends Time

Peter Watson



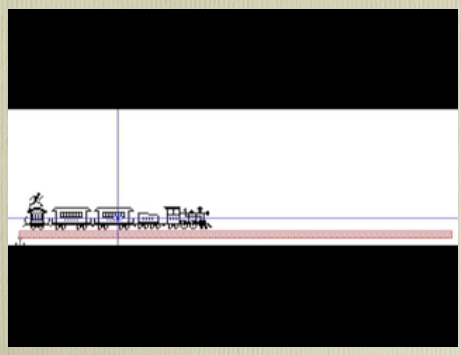
Statutory Warning

- This lecture is for mature audiences only
- Extreme violence may be caused to your pre-conceptions

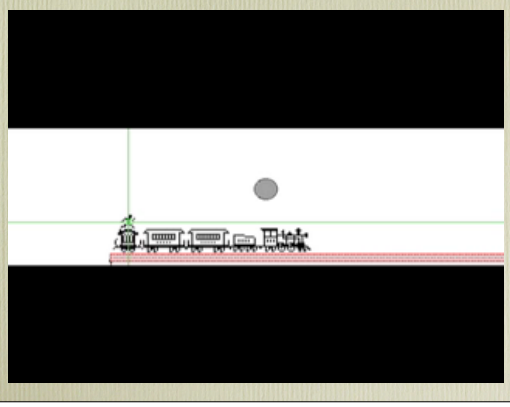
Relative Motion

- Suppose a train is travelling at 5 m/s and a bandit is running towards the front at 2 m/s, relative to the train.
- How fast is he moving relative to the ground?

How fast is he moving relative to the train?



How fast is the ground moving relative to him?



Frames of Reference

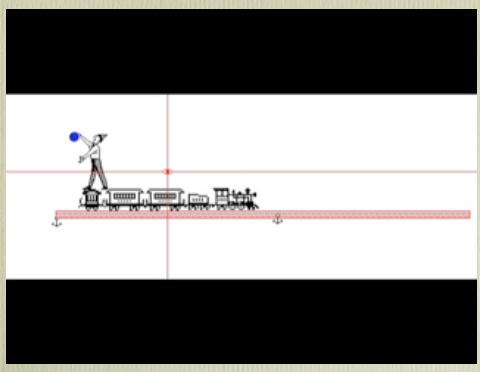
The proper name for "point of view" is "frame of reference": a non-accelerating frame is an "inertial frame"

- This is Galilean Relativity: All inertial frames are equivalent
- Suppose we do experiment in two different frames:

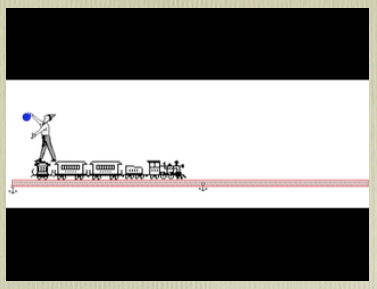
1. Earth Frame: if we measure a distance(velocity) in this frame, we will call it x(v)
2. Train Frame: if we measure a distance(velocity) in this frame, we will call it x'(v')

e.g. just dropping a ball

- In the train frame



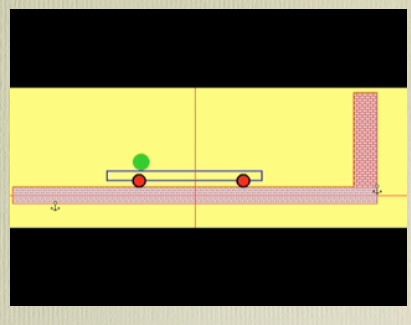
- In the earth frame



- Results of any experiment can be described in any frame: no frame is preferred.
- Put differently: you cannot do an experiment to decide if you are moving, since one man's motion is another man's station!.

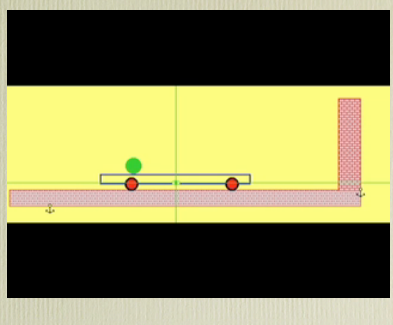
Inertial Frames

- An inertial frame is one that does not accelerate
- Stationary objects stay stationary

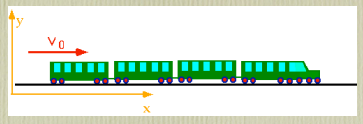


Non-inertial Frames

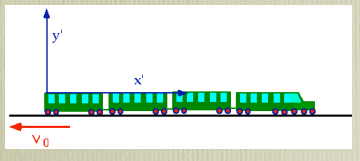
- An non-inertial frame does accelerate
- Stationary objects can accelerate without forces



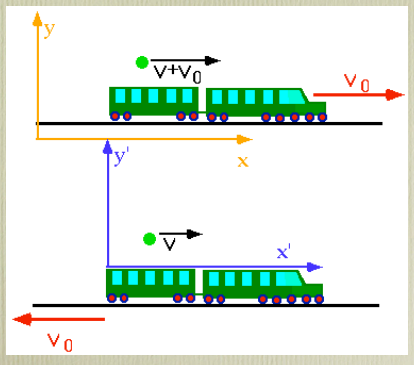
- Can transform the results of an experiment in any one frame to any other.



- Velocity in earth frame = Velocity of train frame + Velocity in train frame
- $v = v_0 + v'$



and can compare them both



Have gone through this in (sordid) detail since it is wrong!

- We have assumed:

1.Laws of Physics are the same in all inertial frames,

2.Time is the same in all frames

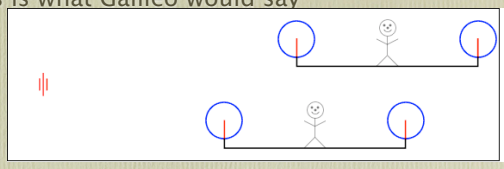
- 2.is a hidden assumption, that was never written down.
- The correct statement (Einstein) is

1.Laws of Physics are the same in all inertial frames,

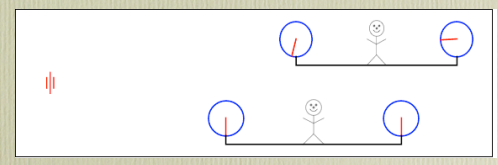
2.The speed of light is the same in all frames

This means that (since speed = distance/time) distance and/or time must change when we go from one frame to another.

- This is what Galileo would say



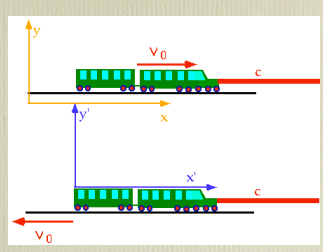
- And this is what Einstein would say



Means clocks must measure different times

Suppose we fire a beam of light from the front of a train.

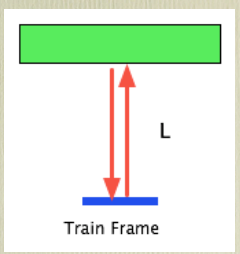
- From the point of view of the earth we would expect
- $c=v_0+c'$
- in fact
- $c=c'$



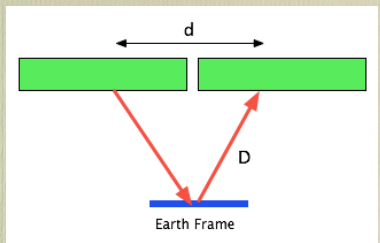
Time Dilation

- To find out how the time changes from one frame to another, consider bouncing a light off a mirror as the train goes past.

$$t = \frac{L}{c}$$



- In the earth frame, the light has to travel further, since the train has moved.



We can solve this

- giving

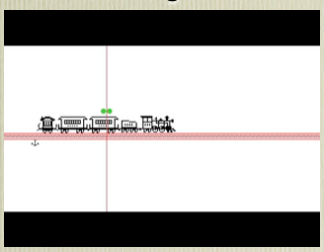
$$t' = t\sqrt{1-\frac{v^2}{c^2}}$$

- so $t'<t$

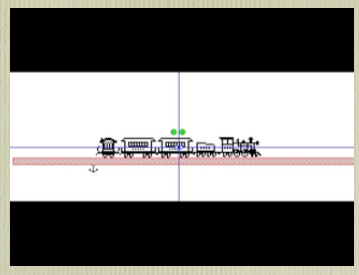
i.e. moving clocks run slow

Simultaneity

- Since time is not the same in two frames, events which are simultaneous in one frame are not in another
- e.g suppose a flash of light is emitted at the centre of a train: when does it get to the end?
- in the earth frame



- but in the train frame



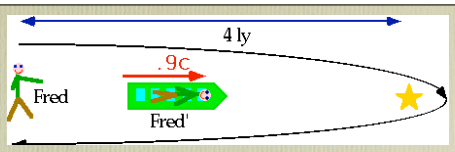
The Twin Paradox

- How much does this slowing down of time matter?
- e.g. Suppose you are in an OC Transpo bus (v_0 = 10ms^{-1}):
- how slow will your watch appear to run compared to your clock at home?
- T = 1 hour at home
- Corresponds to 1 hour - 1 picosecond on the bus
- Note that this correction term is tiny for all cases we are familiar with (which is just as well!)

Note there are a lot of other consequences

- Length contraction (moving objects appear to be shorter)
- Increase of mass (objects get heavier the faster they go, so cannot go faster than light)
- and
- $E = mc^2$

Twin Paradox



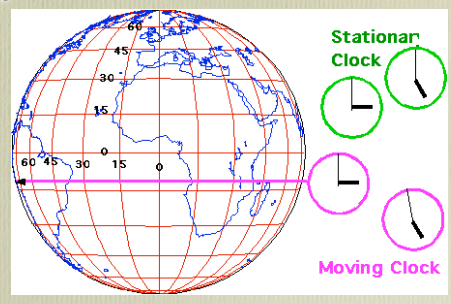
- The star α-Centauri is 4 light-years distant from earth.
- Fred and Fred' are both 20.
- Fred' leaves for α-Centauri at .9 c.
- How old is Fred when Fred' gets back?
- 28.89 yrs
- How old is Fred'?
- 23.87 yrs

Your reaction to all this should be:

- **"This is really stupid. What really happens?"**
- Answer: In physics you cannot ask
- "What really happens?"
- The best one can do is ask
- "What can I measure?"
- Reality is a dangerous concept

So can we measure it?

- Hafele-Keating experiment done in 1980:
- atomic clock flown round the world (first-class!) and compared to time of atomic clock "at rest".
- Time lost by moving clocks ~



Vladimir: That passed the time
Estragon: It would have passed in any case.
Vladimir: Yes, but not so quickly.
Beckett: waiting for Godot.

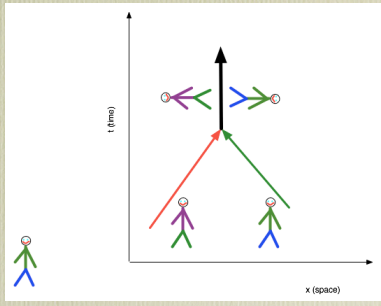
Time as a fourth dimension

The changes to space and time that Einstein found show that they are aspects of the same thing: space-time.

- Galileo-Newton Space is 3-D, time is an independent quantity
- Einstein-Minkowski, Space-time is 4-D, and motion mixes space and time in different ways

- Einstein's concept of time can be expressed graphically by "worldlines" in a space-time diagram.

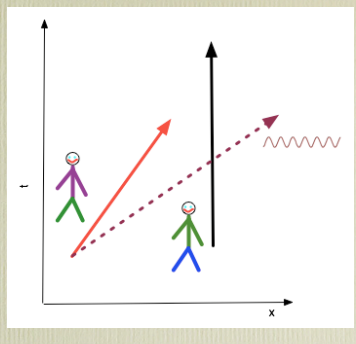
- Reduced to 1 space and 1 time dimension, can describe interactions as events: e.g 2 men walk into each and fall over.



Following constraints must be satisfied by world-lines:

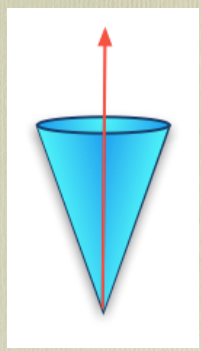
Must be oriented from past to future: "flow of time".

- Static object remains at same x, but time still moves.
- Moving objects have maximum slope corresponding to speed of light.
- Events occur when worldlines intersect

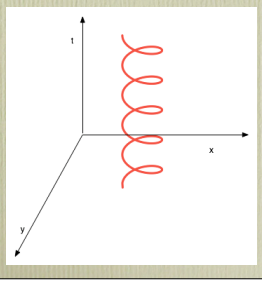


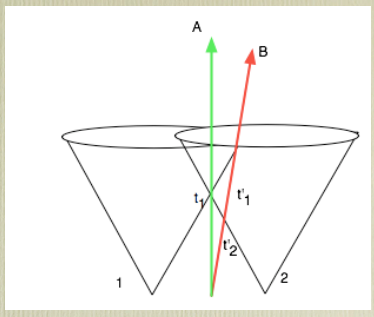
Light Cone

- Possible light paths are represented by "Light Cone".
- Cannot escape the light cone (it includes all possible futures for you!)



- Can see the violation of simultaneity:
- e.g. two flashes of light are seen as simultaneous by observer A but not by B



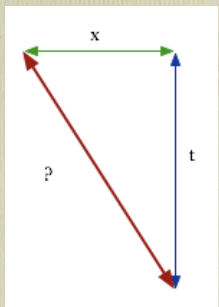


Some world-lines are more complex: e.g. a planet with 2 space dimensions

Statutory Warning: We have represented time as a 4th dimension: this does not mean it **is** the fourth dimension.

- e.g suppose we have an event now and one in the future at time t and position x: the distance is **not**

$$s^2 = x^2 + t^2$$



(in fact we can't even add space and time).

- We must use

$$s^2 = x^2 - c^2t^2$$

- (note the minus sign)
- but even this needs careful interpretation.

e.g. Suppose we send a flash of light:

How does time move in the frame of the light?

- It doesn't: there is no time in this frame.
- Can we describe this in English?
- Imagine you are a photon
- You can't

How about a novel?

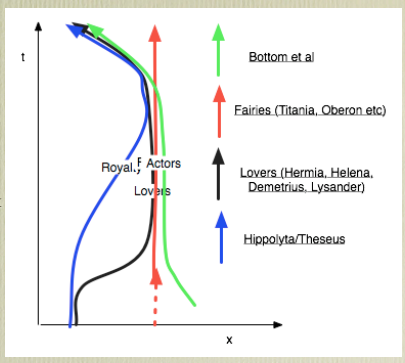
My Life as a Photon

By

Bit Delight

However this 4-D picture is useful. We can analyze the time in any creative work in the same way:

- e.g. Midsummer Nights Dream.
- Note this is a gross over-simplification:
- e.g the lovers + fairies + Bottom have very complex crossing world-lines
- I'll put a girdle round the earth in forty minutes (Puck)



The assertion: prior to 1900 the space-time diagram for any work satisfied the standard conditions: Aristotle's three unities become "space-time causality is preserved" or "special relativity is satisfied".

- Einstein's next question as
- Why do all masses fall at same rate?
- All normal forces (e.g. electrical, friction, elastic...) don't produce same acceleration in all bodies.

$$F = m_I a$$

- The inertial mass m_I) measures how hard things are to accelerate (2nd. law)

- But the gravitational mass (m_G) measures gravitational force or weight

$$F = m_G g$$

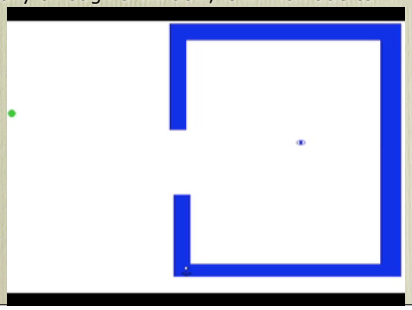
- but we know everything falls at the same rate (well, in a vacuum) so a = g only if the "inertial mass" ≡ "gravitational mass".
- so

$$m_I \equiv m_G$$

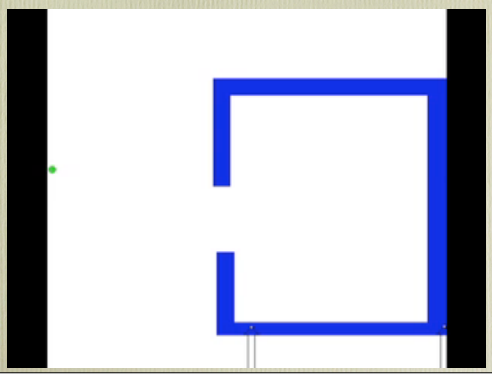
- Are we really sure the m's are the same? This concerned Newton.
- Can demonstrate this is true to 1 part in a trillion (10^{12}) (Eötvos experiment)

Special relativity said you cannot do an experiment to decide if you are moving.

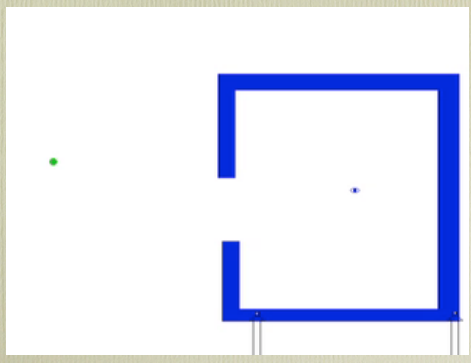
- General says that you cannot do an experiment to distinguish between a gravitational field and an acceleration (!!!!!!!!!)
- Suppose you are in a stationary elevator, and a bullet is shot horizontally through a window, it will fall due to gravity..



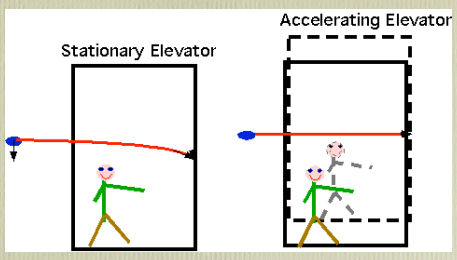
- Suppose you are in an accelerating elevator, and a bullet is shot horizontally, it will travel in a straight line (but the elevator will move up)



- so it will appear to fall inside the elevator.



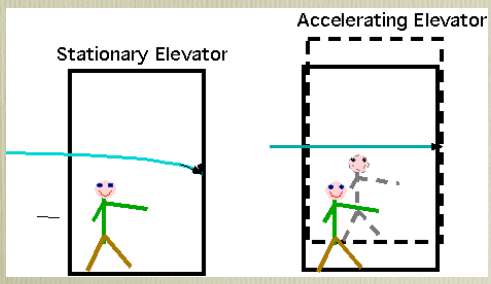
You cannot distinguish them



- This is known as the equivalence principle

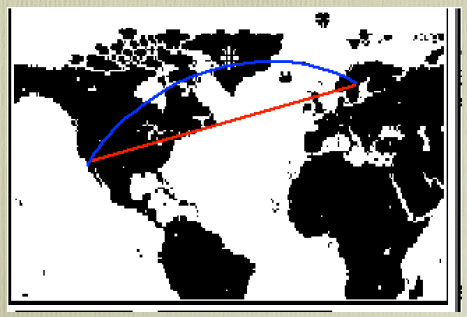
Suppose you are in an accelerating elevator, and a beam of light is shot horizontally, it will appear to fall..

- Suppose you are in a stationary elevator in a gravitational field, and a beam of light is shot horizontally, it will fall..
- You cannot distinguish the two. Light gets affected by gravity?



General relativity:

- What is a straight line?
- A Socratic dialog.....



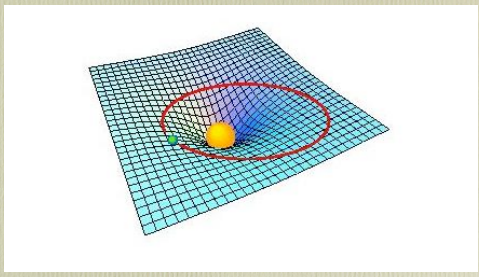
Which is the straight line?

- **A Body continues at rest or in a state of uniform motion unless acted on by a force.**

Uniform motion means in a straight line.

- .....But we are in a curved space .....
- Need a new word: **Geodesic**
- A geodesic in Euclidean space ≡ straight line ≡ shortest path
- **Can either say:**
- 1. There is a force called gravity which acts on all energies (and hence attracts light)
- 2. There is no such thing as gravity, it's just that masses distort space-time in their neighbourhood
- Either way, don't jump off tall buildings: you can be just as dead in a curved space!

Massive bodies follow timelike geodesics so planets are actually moving in "straight" lines in a curved space...

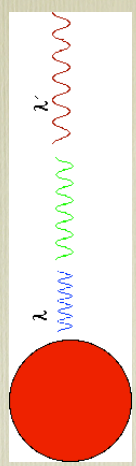


- "Lenses extend unwish through curving wherewhon till unwish returns on its unself" e.e.cummings

Time and Gravity

A ball thrown up near the earth's surface will lose energy.

- How about light?
- Back the the elevator: Light emitted from floor hits ceiling after a time $t = h/c$.
- During this time, lab (elevator) has accelerated to a speed $u = gt$,
- so the light gets stretched out
- But light is a clock ....
- Implies that clocks run slow in gravitational fields



Gravitational Red-shift

- This is another consequence of the equivalence principle:
- confirmed in numerous experiments over the last 40 years, starting with Pound-Rebka
- Means clocks at Earth's surface run slow by ~ .7 ns per second

Gravitational Red-shift

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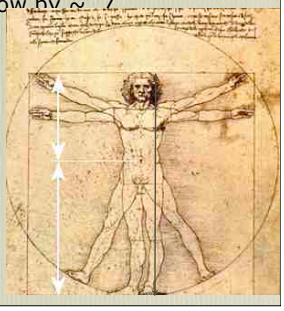
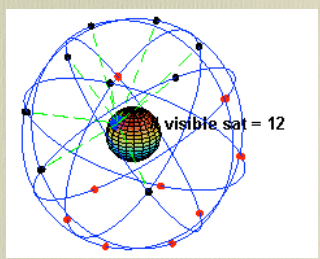
- difference in time over height h is
- $\delta t = gh/c^2$
- so 10^{-16} secs/m
- Can just get this with next generation clocks!

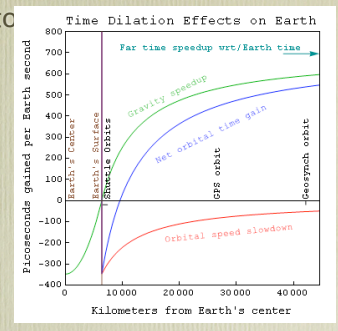


GPS

- needs to be corrected for relativity
- 3 separate effects:
- Sagnac effect: earth rotates, so is not an inertial frame, so events are not simultaneous: can eliminate by using satellites to E and W



- Special relativity: satellite clock is moving relative to earth, so slows down ~ 10^{-10} or 7 μs/day
- GR: satellite clock is in free fall, so speeds up ~ 5×10^{-10} or 46 μs/day
- Would give an 11.7 km error after one day!



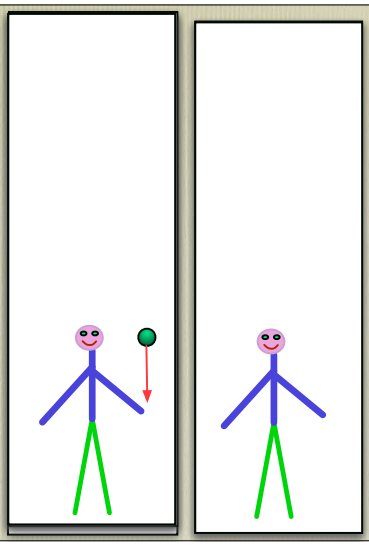
- But suppose gravity was **REALLY** strong.....

Black Holes

- Invented by .....?
- Einstein
- Hawking?
- Well, actually, John Michell, rector of Thornhill Church in Yorkshire
- geologist?philosopher? astronomer? Seismologist?
- Polymath.
- presented his ideas to the Royal Society in London in 1783.

Text

- A particle will escape from the earth if it has positive energy
- At the earth's surface, "escape velocity" is 11 km/s



- A particle will escape from the earth if it has positive energy ➠ v~ 11 km/s
- Turn this: what radius would the earth have for a given escape velocity? If the escape velocity is the speed of light c, nothing can escape

$$R = \frac{2GM}{c^2}$$

- Statutory Warning:This is a fudge: you cannot treat light as a massive particle, nor can you handle a very strong gravitational field as if it were a weak one..... (there are actually two factors of 2 error which cancel out.....weren't we lucky!)

Black Holes

$$R = \frac{2GM}{c^2}$$

- This is the Schwarzchild radius (loosely the black-hole radius) for any mass.
- What is this for the earth?
- ~ 9 mm
- Statutory Warning:This is a fudge: you cannot treat light as a massive particle, nor can you handle a very strong gravitational field as if it were a weak one.....
  (there are actually two factors of 2 error which cancel out.....weren't we lucky!)

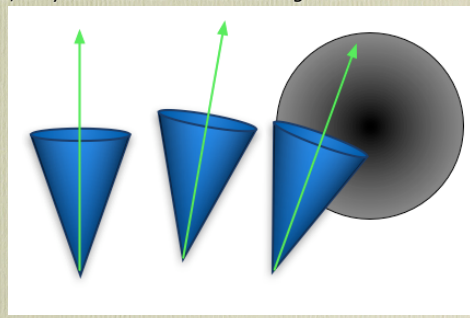
- If the earth was 9 mm in radius, it would be a Black hole
- This is the Schw hole radius

Peter Watson

What happens to time near a black hole

- Gravity modifies the light cone
- close to a black hole, all your futures include falling into it!

A consequence: time stops at the edge of a black-hole for an external observer.



Note that we can still just escape the BH if we move fast enough

Alice never could quite make out, in thinking it over afterwards, how it was that they began: all she remembers is, that they were running hand in hand, and the Queen went so fast that it was all she could do to keep up with her: and still the Queen kept crying 'Faster! Faster!' but Alice felt she COULD NOT go faster, though she had not breath left to say so..... .....

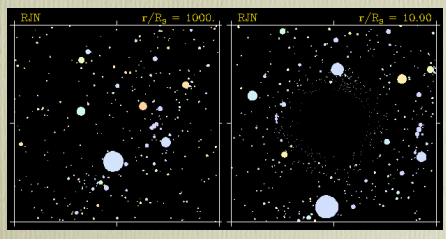
Alice looked round her in great surprise. 'Why, I do believe we've been under this tree the whole time! Everything's just as it was!

"Of course it is,' said the Queen, 'what would you have it?"

"Well, in OUR country,' said Alice, still panting a little, 'you'd generally get to somewhere else--if you ran very fast for a long time, as we've been doing.'

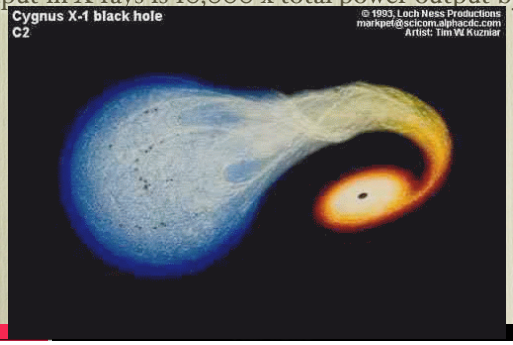
'A slow sort of country!' said the Queen. 'Now, HERE, you see, it takes all the running YOU can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!'

- One way to see a black hole: it's black!
- If we are really lucky...(or unlucky) as a gap in the sky

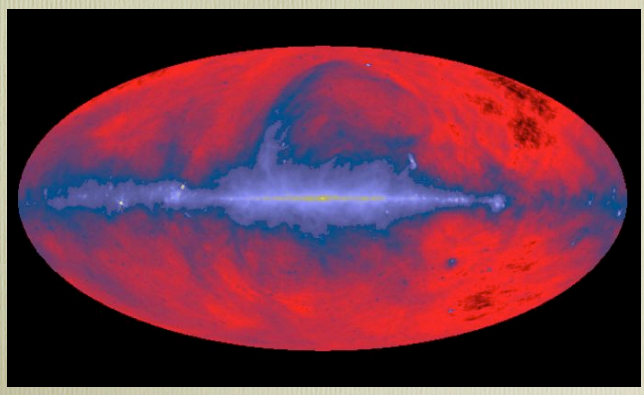


Too Close to a Black Hole
Credit & Copyright: Robert Nemiroff (MTU)<

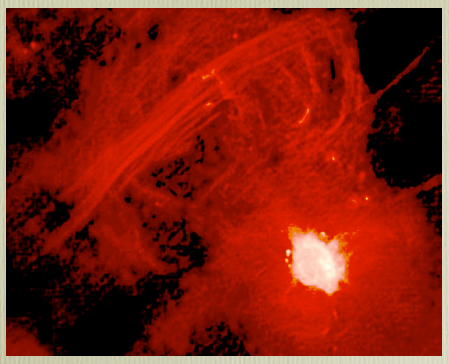
- Stuff falling in will become very hot and produce X-rays
- Cygnus X-1: visible star ~20 mass of sun
- Invisible object M~9$M_\odot$
- Power output in X-rays is 10,000 x total power output by sun!



- If we look at it with radio waves, see very intense source at centre



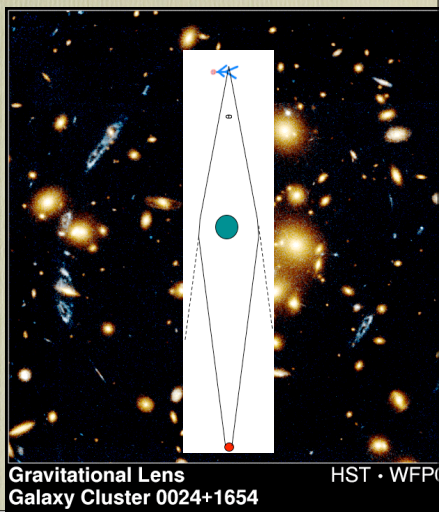
- which gets brighter as we zoom in



- The stars there are swirling round something 10 million times as heavy as the sun

And light does get bent by a massive object

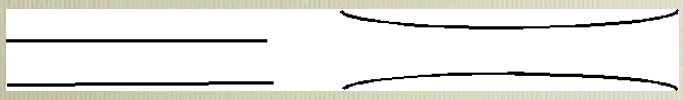
This is a very large cluster of galaxies, which acts as a very large (and rather bad!) lens. It produces several images of a much more distant galaxy



Geometry of Curved spaces

Note we have carefully avoided saying what we mean by a curved space

- Bending of light by gravity allows two (or more) geodesics: i.e. many time-lines connecting same points
- If you take the example of the 2-D curved surface of the Earth, this is embedded in a 3-D space. Hence If a massive body curves space, it almost implies extra dimensions.
- In fact we can carry out tests to decide if we live in a "normal" 3-D space (Euclidean) e.g. parallel lines may be impossible (they get further apart or closer together!)

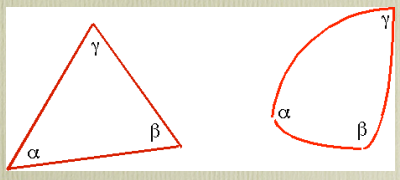


Geometry of Curved spaces

- angles of a triangle add up to 180^0

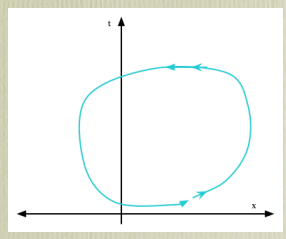
$\alpha + \beta + \gamma = 180$

These are experiments that we can almost do. (Gauss tried the 2nd!).

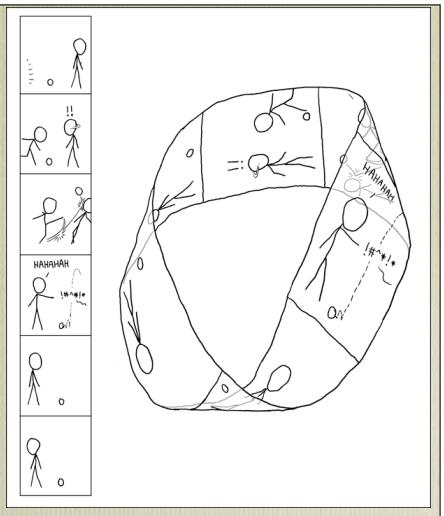


So can we build a time-machine?

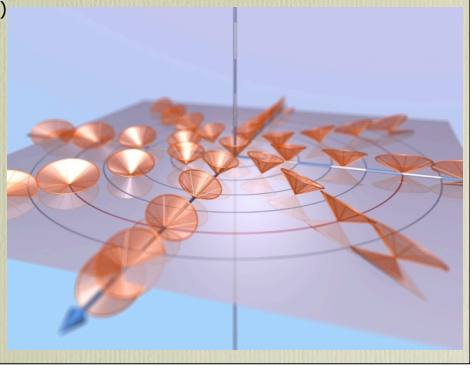
- Now we know the question to ask;
- Can we arrange for world-lines to be closed?



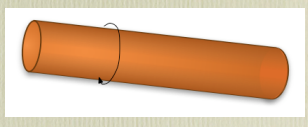
- xkcd.com



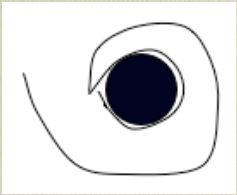
- Godel invented a model universe consistent with GR with closed time-lines. Not like ours:
- it has a centre (ours has no centre)
- it is not homogenous (ours is)
- It rotates (ours doesn't)



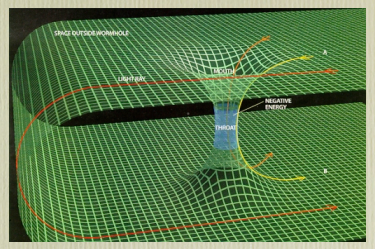
- Godel produced a model universe consistent with GR with closed time-lines
- not like ours: it has a centre is not homogenous and rotates, ours has no centre,seems to be homogeneous and doesn't rotate.
- Tipler showed that can construct time machine from infinite rotating massive cylinder



Light cone gets bent round cylinder, so starting point lies inside light cone



- However once we allow space to be bent, we can construct wormholes!



- Allow instantaneous communication across space
- (And innumerable stupid TV shows)
- But they requires negative energy: now known (see Ford and Roman Sci Am. article) that negative energy allows time-travel, so probably can't construct in practice.

- IF we could time-travel, we run into the paradoxes
- The "Grandfather Paradox"; if I invent a time machine, I can time-travel to the past, murder my grandfather before my father is conceived, so I am not born so I cannot invent the time machine so I cannot .....
- The "Where are they" paradox; if time travel is possible, why aren't we over-run by time tourists?

Let's try to summarize the mess we are in

- We have lost the idea of universal time and with it
  - The concept of simultaneity
  - The concept of a universal "now"
  - The idea that Euclid was right!

Let's try to summarise the mess we are in

- We have gained
  - The linking of time and space into space-time
  - Black holes
  - Curved Spaces
  - Multiple time-lines connecting events
  - Time-travel ??????????????????????
  - The concept of space without time

Why is the speed of light so special?

- It isn't: it's just the maximum speed that anything can move at.
- Anything massless always moves at c (photons, neutrinos almost)
- Anything massive (protons, electrons, spaceships) can approach c but not get there
- It is really a number that relates distance to time

Finally

- A somewhat subtle point
- Originally we had “universal time”
- Not crazy to think that time can change, but then it should be tied to measurement (e.g. clock, photon, biology .....)
- It is now connected to a “frame of reference”, disconnected from any measurement
- Lets do something simpler! Can we at least predict things?!