Grouse, Hurricanes and Dead Cats: How to predict

Peter Watson

Paul Bourke

Prediction and Time

Even if we cannot time-travel, perhaps we can do it "virtually"

if we know exactly how a system works & how it changes with time, we should be able to predict its future.

What can we predict?

Deterministic systems: i.e. systems whose future can be predicted exactly e.g. planetary system, mass on a spring, pendulum.

Random systems: i.e. ones which are too complex to predict exactly e.g. gas, society...Best we can do is to predict average values

However there are two other kinds of systems:

Chaotic: i.e. systems which are predictable over the short term but not over the long term.

Quantum: systems which are intrinsically unpredictable except in a special sense.

Chaotic Motion

She comes, she comes, the sable throne behold
Of Night Primeval and of Chaos old!
...
Physic of Metaphysic begs defence
And Metaphysic calls for aid on Sense
See Mystery to Mathematics fly
In vain! they gaze, turn giddy, rave and die
....
Lo! thy Dread Empire, Chaos is restored
Light dies before thy uncreating Word.
Alexander Pope, The Dunciad

The easiest one to visualize, (technically it is not chaotic), is the "baker transform".

Take a piece of dough with a raisin

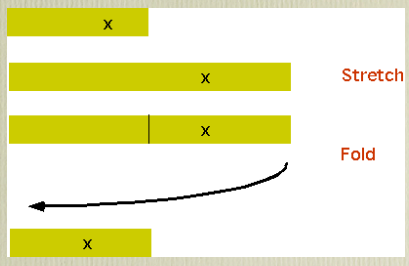
Stretch it to twice it's original length

Fold it in half

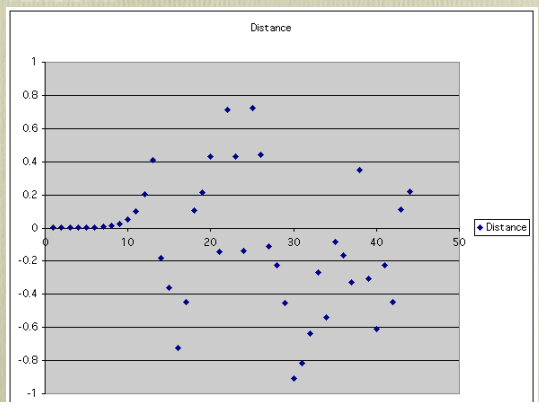
Where is the raisin?

The formula is

$$x_n = 2x_{n-1} \quad (x_{n-1} < .5)$$
$$x_n = 2 - 2x_{n-1} \quad (x_{n-1} > .5)$$



For example, we can start with two raisins very close together and see what happens:
If you plot the difference in their positions , it looks nice and smooth to start with, but suddenly becomes random.



in Arcadia, Valentin wonders why the population of grouse on the moors isn't predictable. Suppose we have a population which grows

No. of births ∝ No of live individuals

$$x_{birth} = kx_{live}$$

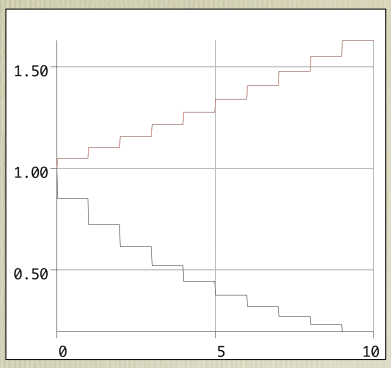
No. of deaths ∝ no. of live individual

$$x_{death} = k' x_{live}$$

So total number in next generation is

$$x_{n+1} = x_n - x_{death} + x_{birth}$$
$$= (1 + k - k')x_n$$

If this is all, population grows (or dies!) exponentially.



But suppose we add in starvation

- In that case if the population grows too large, there will be starvation, and this deaths will increase more rapidly: say as square of the population

$$x_{starve} = k'' x_{live}^2$$

- So total number in next generation is

$$x_{n+1} = x_n - x_{death} - x_{starve} + x_{birth}$$
$$= (1 + k - k')x_n - k'' x_n^2$$

In Practice

- Suppose k = .3, k' = .15, k" = .001

#	Born	Die	Starve	Next #
100	30	15	10	105
200	60	30	40	190

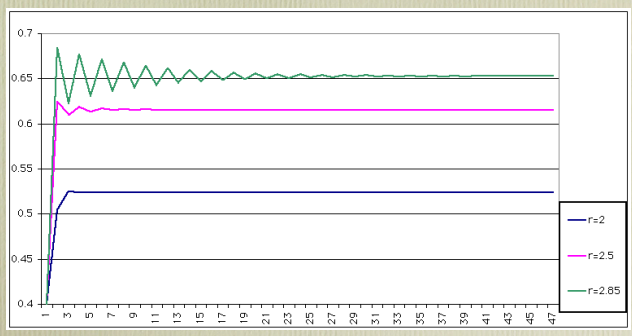
- " Obviously " what will happen is that the population will grow until the population reaches an equilibrium value?

150	45	22	23	150

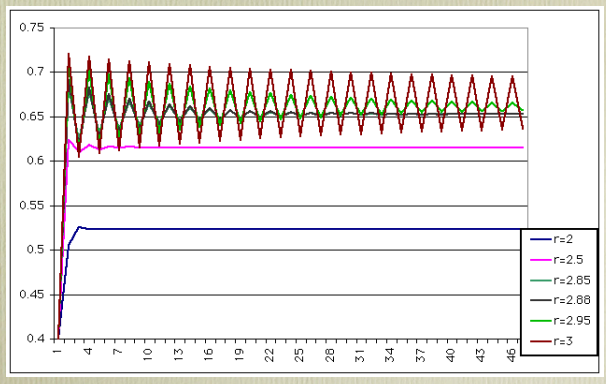
We can make this look a bit cleaner by writing

$$x_{n+1} = rx_n(1 - x_n)$$

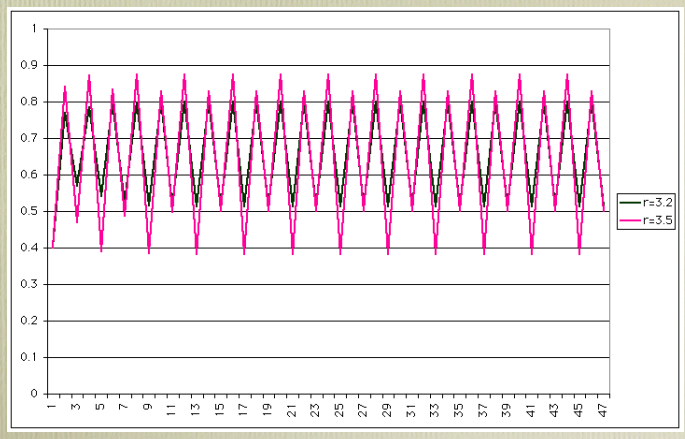
Deaths = Births (but there will be a bit of overshoot)



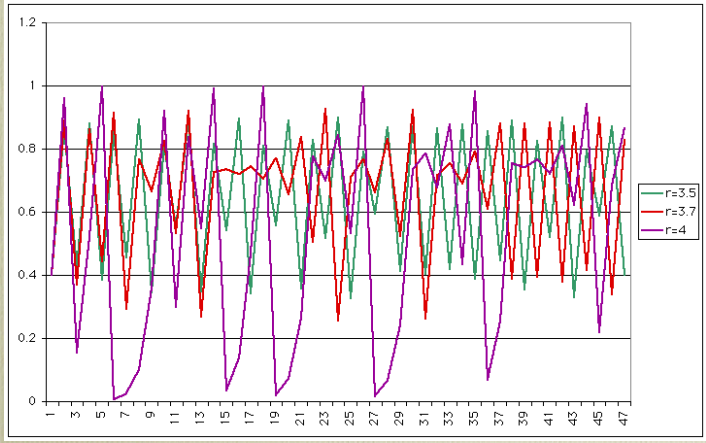
- But then the "overshoot" doesn't die away and the system oscillates



- But then it gets worse



- and worse



Logistic Map

- VALENTINE "You have some x-and-y equations. Any value for x gives you a value for y. So you put a dot where it's right for both x and y. Then you take the next value for x which gives you another value for y, and when you've done that a few times you join up the dots and that's your graph of whatever the equation is....every time she works out a value for y, she's using that as her next value of x. And so on." Arcadia

Chaotic Systems

All chaotic systems have some common features

The equations must all be non-linear: i.e. Have terms like x^2

There are regions of the parameters where the motion is predictable

There are regions where it is chaotic

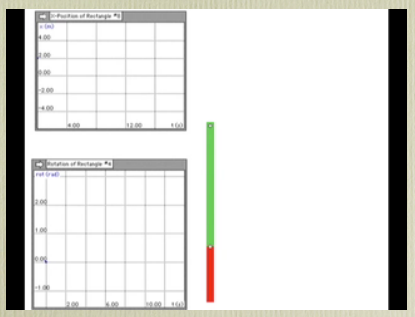
In the chaotic region, points that start off close together become wildly different as time goes on.

Note the importance of non-linearity!

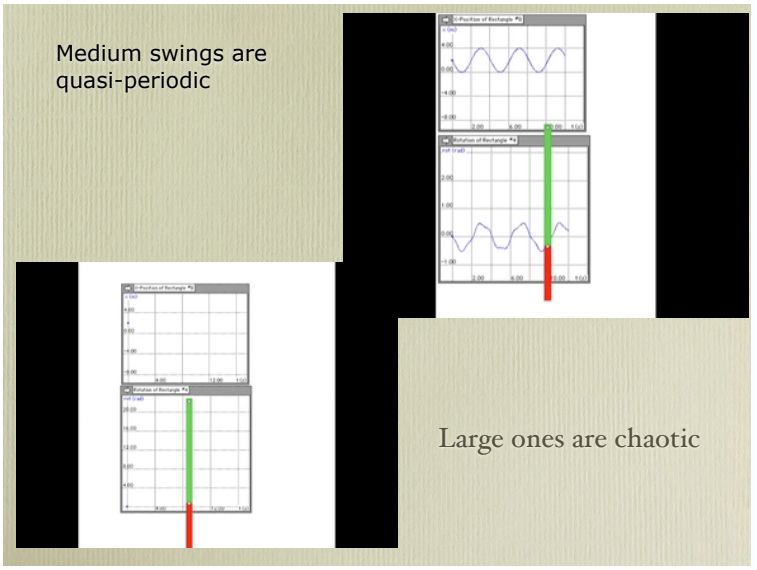
- Linear systems can be unmapped

Double pendulum

- Small swings are predictable,



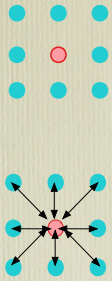
Medium swings are quasi-periodic



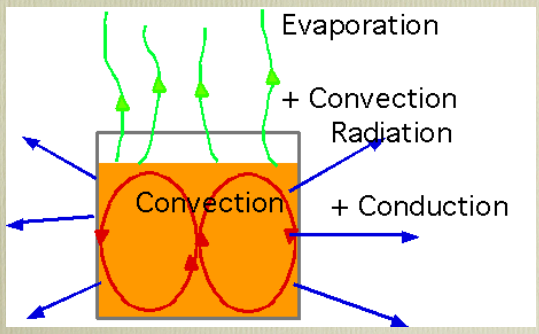
Large ones are chaotic

Weather

- "Primitive Equations" for weather written down by L F Richardson (1922). Can't be solved without computer
- Assume we know everything (temperature, pressure, humidity, radiation inflow...) at some points in space.
- Each point will affect it's neighbour, so can figure out how it will change
- Need to know how the energy can be transferred



- Note that all these processes work together
- e.g your coffee!



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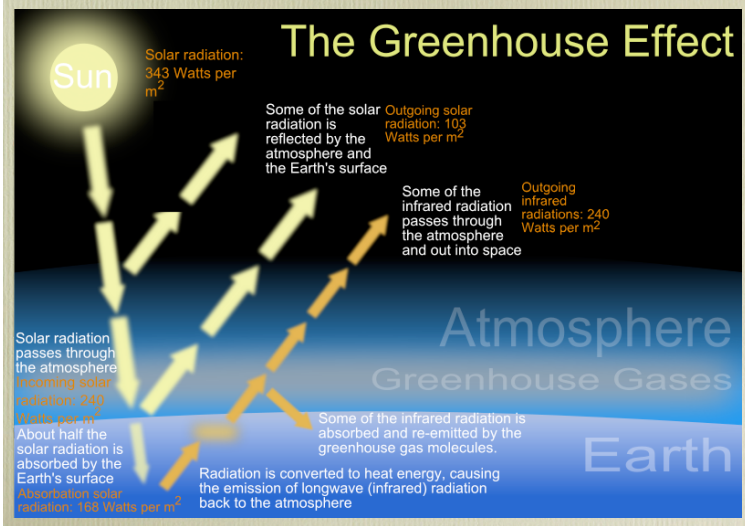
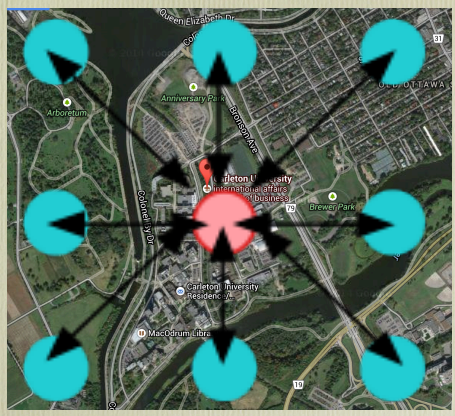
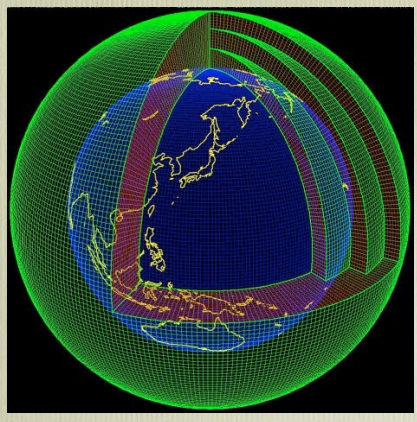




This is how we do it



But

Butterfly effect found in 1950's: arbitrarily small perturbation of initial conditions have unpredictably large consequences.

The "Lorentz" equations: very simplified version of the "weather " equations, give rise to chaotic behaviour.

Weather is also chaotic

You cannot predict the future weather precisely.

However, buried in this are some predictable elements. e.g. we cannot predict an "el Nino" event, but we can predict the consequences once it has happened.

Note "weather" prediction and "climate" prediction are (almost) unrelated

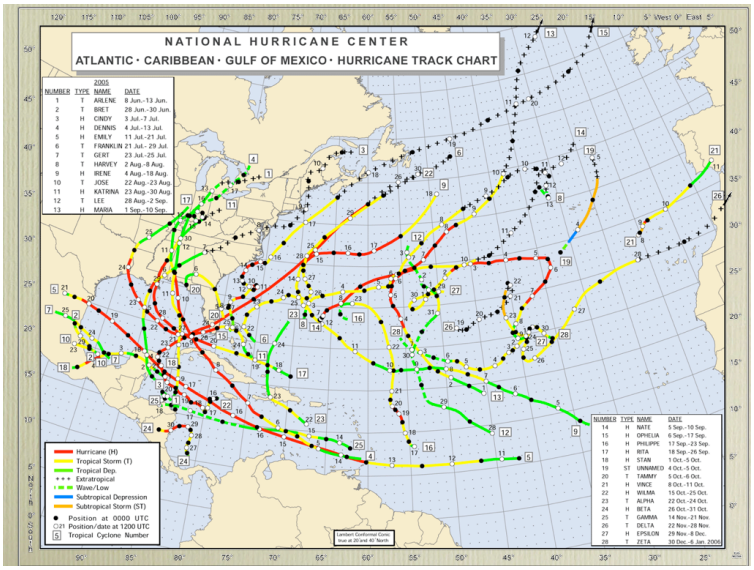
•Can predict globally, not locally
Can predict how fast a river will flow

- But not how it will behave on small scale

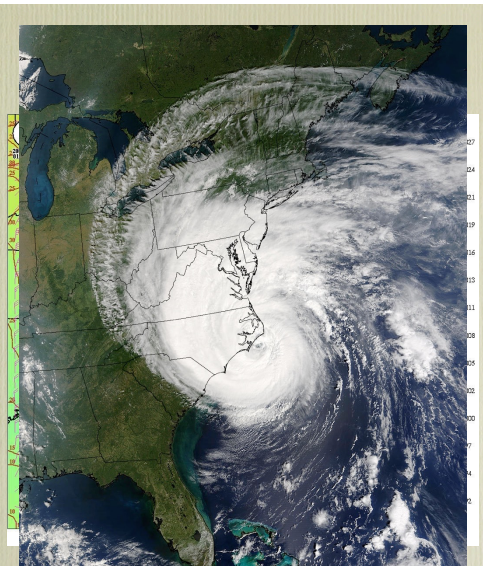
PW

Hurricanes

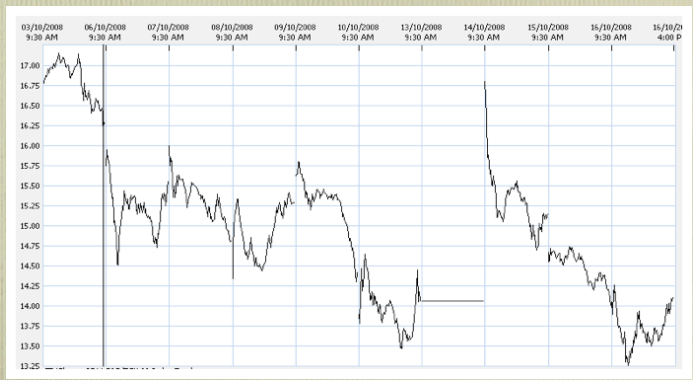
NASA picture



Can do it over the short term Hurricane Isabelle



An interesting chaotic system (provided your pension doesn't depend on it!)



- At the start of the crisis financial firms held huge dollops of each others equity..Such tight coupling increases the danger of "non-linear" outcomes, where a small change has a big impact. Economist Feb 2010

Now we do the hard stuff

- Quantum Mechanics
- I think I can safely say that nobody understands quantum mechanics. (Richard Feynman.)

What is light?

Particle? Newton, Descartes
Kerner: Look at the edge of the shadow. It is straight like the edge of the wall that makes it. This means light is ..little bullets. Bullets go straight.
Hapgood (Tom Stoppard)

Wave? Young, Huyghens
Kerner: When you shine a light through two little gaps, side by side, you don't get particle patterns like for bullets, you get wave patterns like for water. The two beams of light mix together
Hapgood (Tom Stoppard)

Yes? Planck/Einstein
Light travels as wave, but arrives and departs as particle

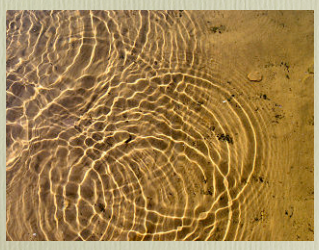
Wave-Particle Duality

De Broglie 1924

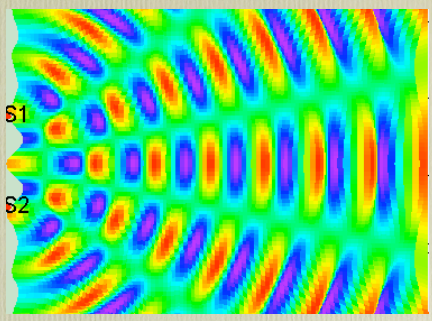
- You cannot ask: Is light a wave or a particle: answer is "yes"
- so maybe electron (particle) has some wave properties
- All fundamental (i.e small!) particles also act like waves (what is an electron?...) and waves act like particles.

Waves in General

- Can show "interference" : sometimes waves will add together, sometimes cancel out

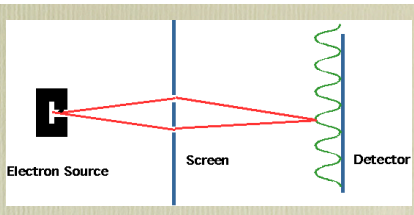


- Like this



Wikisource

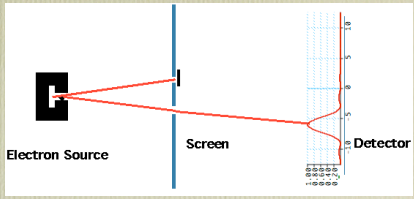
- We can now do this with electrons: Very low energy electrons pass through slits and hit detector (e.g. photo plate) and give 2-slit interference pattern



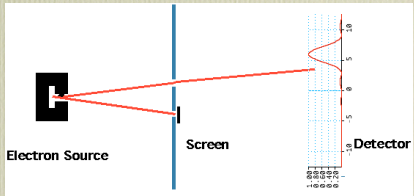
You can even watch how it builds up, one electron at a time

- The electron is a particle, with charge. It must go through one slit or the other...

Suppose we close off one slit:



Suppose we close the other slit:



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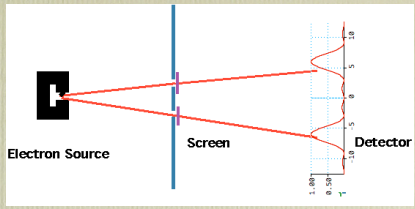
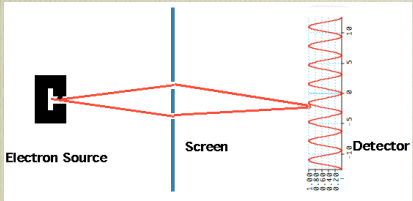
When we add together two one slit patterns, We get this



Not what we get from 2-slits together



PW

- Suppose we get sneaky and allow electron through but check which slit it went through.

Now we get sum of one slit patterns, but not a 2 slit pattern!

More worrying than this: we can do a "delayed choice" experiment: don't try to observe the electron until **after** it has gone through one of the slits...that still destroys the pattern.

Conclusion **We cannot decide which slit the electron went through without destroying the pattern. Observing something fundamentally changes it!**

Kerner: Now we come to the exciting part. We will watch the bullet to see how they make waves ...The wave pattern has disappeared
Because we looked. Every time we don't look, we get wave pattern. Every time we look to see how we get wave pattern we get particle pattern
Hapgood (Tom Stoppard)

There was a young man who said "God
Must think it exceedingly odd
That this tree
Continues to be
When there's no one about in the Quad"

So why should you care, since this is a lecture about Time?

- Because we cannot say what happened **after** it happened!

Measurement

In classical mechanics, we believe that a object is the same whether we measure it or not.

In quantum mechanics, until we have measured it, its condition is indeterminate.

E.g.: suppose we measure the position of a parti and it was here → C

- Where was it just before?
- **Classical Mechanic** At C.
- **Quantum Mechanic** Somewhere: it was only measuring it that fixed its position . Where is a candle flame after it is blown out?

Have we given free will to the electron?

When did the electron decide which slit it went through?

- **Classical Mechanic** Obviously at the moment it was hit the slits.

  Quantum Mechanic It is indeterminate until you measure it

- The Einstein–Podolsky–Rosen paradox (EPR) is a more sophisticated version of this

  God does not play dice. Einstein

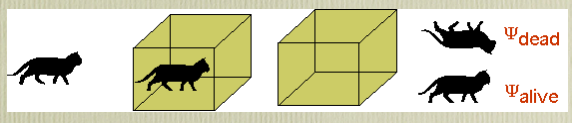
  Only way out is "hidden variables": underneath quantum mechanics, there is some “clockwork”. it only looks random on the surface.

Schrödinger's Cat

was supposed to show the idiocy of people who really believed in quantum mechanics.

You have a box, with a lid and a single radioactive atom: when the atom decays, cyanide gas is released.

- Take a cat
- Put it in the box and close the lid.
- Is the cat dead or alive?



- **Classical Mechanic** Obviously its either dead or alive
- **Quantum Mechanic** It is indeterminate until you measure it . More exactly, the cat is a mixture of alive and dead cats: the measurement fixes it.
- **Schrödinger** Don't be stupid.

Both Einstein and Schrödinger were wrong.

Bell's theorem shows that there is a measurement that you can do on the polarizations of the particles which is incompatible with any possible hidden variable theory.

Aspect did the experiment.

The Schrödinger's Cat experiment has been done:

No animals were injured in the making of this movie.

One atom: process is totally random, so you can't decide if a one-atom cat is alive or dead without measuring it(!)

Many atoms (10^{29}): constitutes an independent measuring system, so the cat measures it's own deadness

Few atoms (2-20): process becomes steadily more predictable

God not only plays dice, but throws them where they cannot be seen. Hawking

- We can calculate measured values with phenomenal accuracy
- E.g. An electron acts like a tiny magnet: exactly how tiny?
- In sensible units
- -1.001159652181 (2006 measured)
- -1.001159652182 (2008 theory)
- So quantum mechanics cannot be ***wrong***

Measurement

- This "measurement fixes things" is known as the "Collapse of wave function": obviously very ugly .

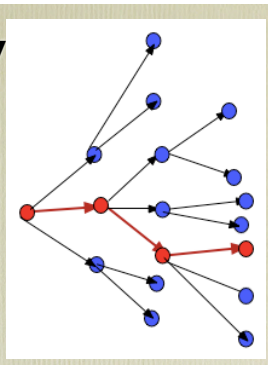
How does the electron know it is being measured?.

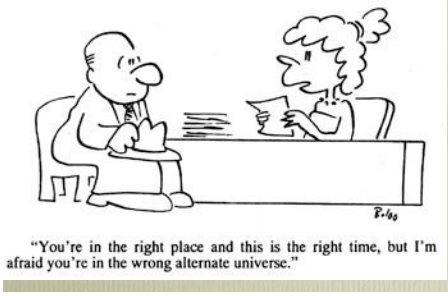
Do we need an actual conscious observer?

Is there a link between consciousness and QM?

Many worlds theory

Everett (1957) . Every time a measurement is made, the universe subdivides into separate universes that correspond to every possible outcome





"You're in the right place and this is the right time, but I'm afraid you're in the wrong alternate universe."

Avoids observation problems, but not testable (?) and not very economical!

In all fictional works, each time a man is confronted with several alternatives, he chooses one and eliminates the others; in the fiction of Ts'ui Pên, he chooses-simultaneously-- all of them. He creates in the diverse way, diverse futures..which themselves also proliferate and fork.

The Garden of Forking Paths, Borges.

What might have been is an abstraction
Remaining a perpetual possibility
Only in a world of speculation.
What might have been and what has been
Point to one end, which is always present.
Footfalls echo in the memory
Down the passage which we did not take
Towards the door we never opened
Into the rose-garden.
T. S. Eliot (Burnt Norton)

Conclusions:

Either Quantum mechanics is correct, and there is no "simpler" system

Or Reality is even uglier than we thought: e.g.

non-local hidden variables: every bit of the universe is involved with every other bit:

very Zen, but totally wipes out free will!

???????????

(Ugh!)

Conclusions:

Does it bother you that 20th century technology depends fundamentally on something no-one understands?

I can only say, **there** we have been: but I cannot say where.

And I cannot say, how long, for that is to place it in time.

T. S. Eliot (Burnt Norton)